

Chapter 3—parallel and perpendicular lines

You will be expected to write some simple proofs on your own by the end of this chapter. You want to try some on your own—not just copy them from class. It gets a lot easier with practice. You do not want your first attempt to be during a test.

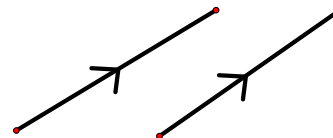
- **Section 1**

- Lines that do not intersect

- Parallel = _____

The symbol for the word parallel is _____.

The picture will show that lines are parallel.



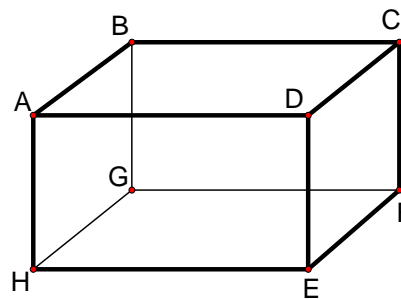
‘Lines that do not intersect’ is NOT an adequate definition for the word parallel.

- Skew = _____

There is no symbol for skew.

Use the picture to answer the following.

- _____ is a line parallel to \overline{AB}
- _____ is a line perpendicular to \overline{AB}
- _____ is a line skew with \overline{AB}
- _____ is a line that intersects \overline{AB}



Parallel planes = _____

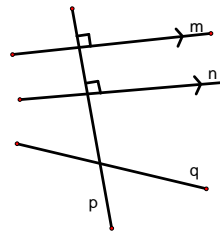
Name a line that appears \parallel to \overline{PQ} and contains S	
Name a line that appears \perp to \overline{PQ} and contains S	
Name a line skew to \overline{PQ} and contains S	
plane PRQ \parallel plane _____	
plane PRQ \perp plane _____	
plane URQ intersects plane _____ at \overline{TQ}	
plane PSQ intersects plane PRU at _____	

Fill in each blank with \parallel , \perp , or neither

lines n and m are _____

lines p and q are _____

lines n and p are _____



PARALLEL POSTULATE

If there is a line and a point not on the line, then there is _____ line through the point that is \parallel to the given line.

PERPENDICULAR POSTULATE

If there is a line and a point not on the line, then there is _____ line through the point \perp to the given line.

Transversal = any line that _____

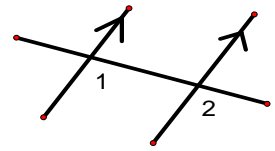
<p>Corresponding Angles $\angle 1$ and \angle____ $\angle 7$ and \angle____ $\angle 2$ and \angle____ \angle____ and \angle____</p>	
<p>Alternate Exterior Angles $\angle 1$ and \angle____ $\angle 4$ and \angle____</p>	
<p>Alternate Interior Angles $\angle 2$ and \angle____ $\angle 3$ and \angle____</p>	
<p>Consecutive Interior Angles (a.k.a. Same-side Interior Angles) $\angle 2$ and \angle____ \angle____ and \angle____</p>	
<p>Linear Pairs of Angles $\angle 1$ and \angle____ $\angle 5$ and \angle____ $\angle 1$ and \angle____ $\angle 5$ and \angle____ $\angle 3$ and \angle____ $\angle 7$ and \angle____ $\angle 3$ and \angle____ $\angle 7$ and \angle____</p>	
<p>Vertical Angles $\angle 1$ and \angle____ $\angle 5$ and \angle____ $\angle 2$ and \angle____ $\angle 6$ and \angle____</p>	

	$\angle 9$ and $\angle 11$ are _____ angles
	$\angle 6$ and $\angle 10$ are _____ angles
	$\angle 8$ and $\angle 11$ are _____ angles
	$\angle 7$ and $\angle 13$ are _____ angles
	$\angle 6$ and $\angle 8$ are _____ angles
	$\angle 10$ and $\angle 11$ are _____ angles

• **Section 2**

• **corresponding angles postulate**

- if 2 \parallel lines are cut by a transversal, then corresponding angles are _____.
(This is a POSTULATE so it is offered without proof.)



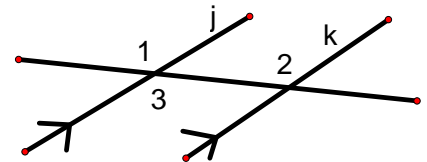
<p>$m \angle 1 = \underline{\hspace{2cm}}$</p>	<p>$m \angle 2 = \underline{\hspace{2cm}}$</p>
<p>$x = \underline{\hspace{2cm}}$</p>	<p>$x = \underline{\hspace{2cm}}$</p>

• **alternate interior angles theorem**

- if 2 \parallel lines are cut by a transversal, then alternate interior angles are _____.

Given: line $j \parallel$ line k

Prove: $\angle 2 \cong \angle 3$



line $j \parallel$ line k

given

$\angle 1 \cong \angle 2$

$\angle 1 \cong \angle 3$

$\angle 2 \cong \angle 3$

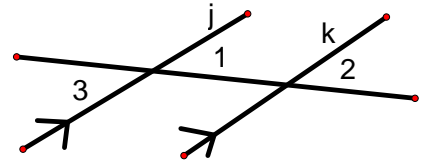
<p>$m \angle 5 = \underline{\hspace{2cm}}$</p>	<p>$m \angle 6 = \underline{\hspace{2cm}}$</p>
<p>$x = \underline{\hspace{2cm}}$</p>	<p>$x = \underline{\hspace{2cm}}$</p>

• **alternate exterior angles theorem**

- if 2 \parallel lines are cut by a transversal, then alternate exterior angles are _____.

Given: line $j \parallel$ line k

Prove: $\angle 2 \cong \angle 3$



line $j \parallel$ line k given
 $\angle 1 \cong \angle 2$ _____
 $\angle 1 \cong \angle 3$ _____
 $\angle 2 \cong \angle 3$ _____

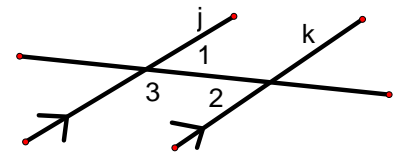
<p>$m\angle 4 =$ _____</p>	<p>$m\angle 5 =$ _____</p>
<p>$x =$ _____</p>	<p>$x =$ _____</p>

• **consecutive interior angles theorem**

○ if 2 \parallel lines are cut by a transversal, then consecutive interior angles are _____.

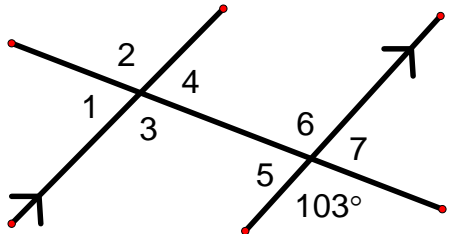
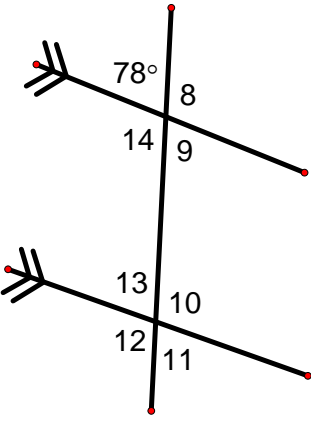
Given: line $j \parallel$ line k

Prove: $m\angle 2 + m\angle 3 = 180^\circ$



line $j \parallel$ line k given
 $\angle 1 \cong \angle 2$ _____
 $m\angle 1 + m\angle 3 = 180^\circ$ _____
 $m\angle \underline{\quad} + m\angle \underline{\quad} = 180^\circ$ _____

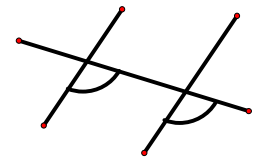
<p>$m\angle 5 =$ _____</p>	<p>$m\angle 6 =$ _____</p>
<p>$x =$ _____</p>	<p>$x =$ _____</p>

 <p> $m\angle 1 = \underline{\hspace{2cm}}$ $m\angle 2 = \underline{\hspace{2cm}}$ $m\angle 3 = \underline{\hspace{2cm}}$ $m\angle 4 = \underline{\hspace{2cm}}$ $m\angle 5 = \underline{\hspace{2cm}}$ $m\angle 6 = \underline{\hspace{2cm}}$ $m\angle 7 = \underline{\hspace{2cm}}$ </p>	 <p> $m\angle 8 = \underline{\hspace{2cm}}$ $m\angle 9 = \underline{\hspace{2cm}}$ $m\angle 10 = \underline{\hspace{2cm}}$ $m\angle 11 = \underline{\hspace{2cm}}$ $m\angle 12 = \underline{\hspace{2cm}}$ $m\angle 13 = \underline{\hspace{2cm}}$ $m\angle 14 = \underline{\hspace{2cm}}$ </p>
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• **Section 3**

Remember that CONVERSE means that the hypothesis and conclusion have been switched.

- Proving that the lines are parallel
 - **Corresponding angles converse postulate**
 - If corresponding angles are _____, then the 2 lines are _____.

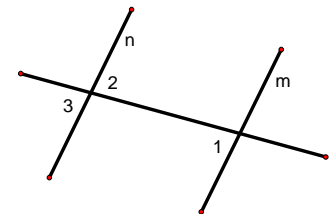


- **Alternate interior angles converse theorem**
 - If alternate interior angles are _____, then the 2 lines are _____.

Given: $\angle 1 \cong \angle 2$

Prove: line $m \parallel$ line n

$\angle 1 \cong \angle 2$ _____
 $\angle 2 \cong \angle \underline{\hspace{1cm}}$ vertical angles are congruent
 $\angle \underline{\hspace{1cm}} \cong \angle \underline{\hspace{1cm}}$ transitive property
 _____ if corresponding angles are \cong , then the lines are _____

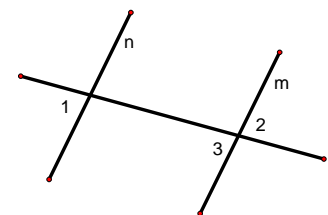


- **Alternate exterior angles converse theorem**
 - If alternate exterior angles are _____, then the 2 lines are _____.

Given: $\angle 1 \cong \angle 2$

Prove: line $m \parallel$ line n

$\angle 1 \cong \angle 2$ _____
 $\angle 2 \cong \angle \underline{\hspace{1cm}}$ vertical angles are congruent
 $\angle \underline{\hspace{1cm}} \cong \angle \underline{\hspace{1cm}}$ transitive property
 _____ if corresponding angles are \cong , then the lines are _____



○ **Consecutive interior angles converse theorem**

- If consecutive interior angles are _____, then the 2 lines are _____.

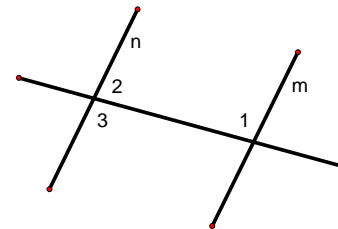
Given: $m\angle 1 + m\angle 2 = 180^\circ$

Prove: line $m \parallel$ line n

 $m\angle 2 + m\angle 3 = \underline{\hspace{2cm}}^\circ$
 $m\angle 1 + m\angle 2 = m\angle \underline{\hspace{1cm}} + m\angle \underline{\hspace{1cm}}$
 $m\angle \underline{\hspace{1cm}} = m\angle \underline{\hspace{1cm}}$

given
 definition of linear pair
 substitution

 alternate interior angles congruent \rightarrow lines are \parallel



These 4 rules are all BICONDITIONAL statements.

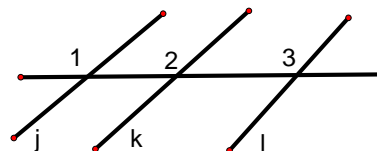
corresponding angles are congruent $\Leftrightarrow \parallel$ lines with a transversal
alternate interior angles are congruent $\Leftrightarrow \parallel$ lines with a transversal
alternate exterior angles are congruent $\Leftrightarrow \parallel$ lines with a transversal
consecutive interior angles are supplementary $\Leftrightarrow \parallel$ lines with a transversal

○ **Transitive property of parallel lines theorem**

- If 2 lines are \parallel to the same line then they are _____

Given: line $j \parallel$ line k
 line $j \parallel$ line l

Prove: line $k \parallel$ line l

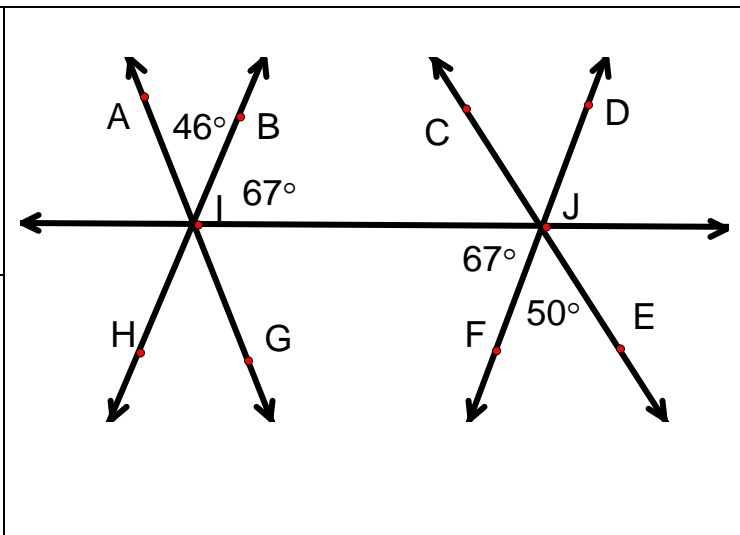


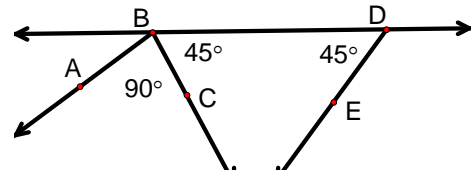
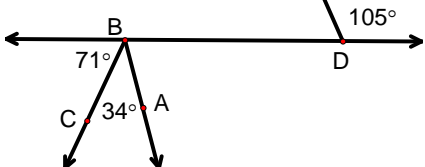
 $\angle 1 \cong \angle 2$

 $\angle \underline{\hspace{1cm}} \cong \angle \underline{\hspace{1cm}}$
 _____ \parallel _____

 given
 corresponding angles are congruent
 substitution
 corresponding angles are congruent

Are \overleftrightarrow{AG} and \overleftrightarrow{CE} parallel?
 Are \overleftrightarrow{BH} and \overleftrightarrow{DF} parallel?



Are \overline{BA} and \overline{DE} parallel?	
Are \overline{BA} and \overline{DE} parallel?	

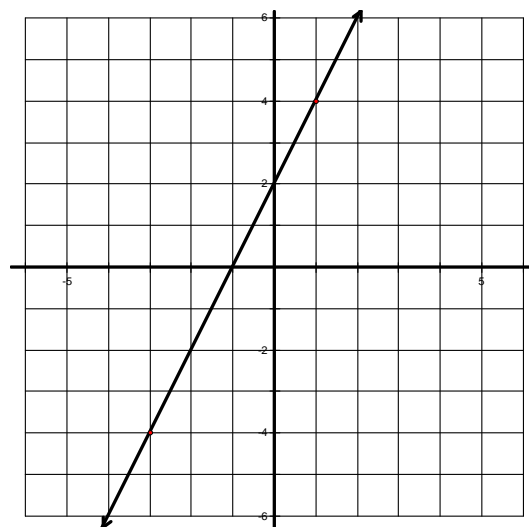
• **Section 4**

Slope = _____

○ SLOPE FORMULA: $m = \frac{\text{rise}}{\text{run}}$

Calculate the slope of the line shown in the graph.

$m = \frac{\text{rise}}{\text{run}} = \frac{\quad}{\quad} = \quad$



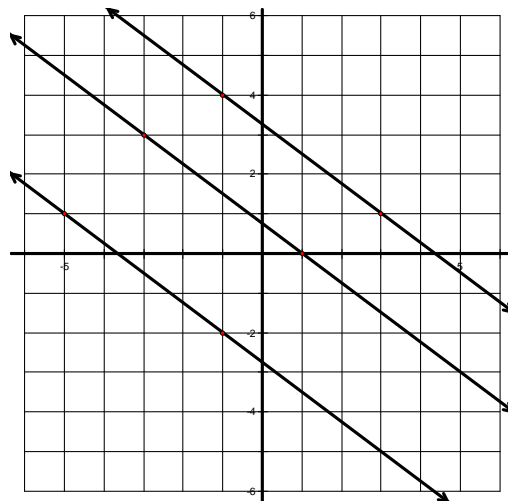
○ **Slopes of parallel lines**

The graph shows several lines that are parallel. Calculate the slope of each of the lines.

$m = \frac{\text{rise}}{\text{run}} = \frac{\quad}{\quad} = \quad$

$m = \frac{\text{rise}}{\text{run}} = \frac{\quad}{\quad} = \quad$

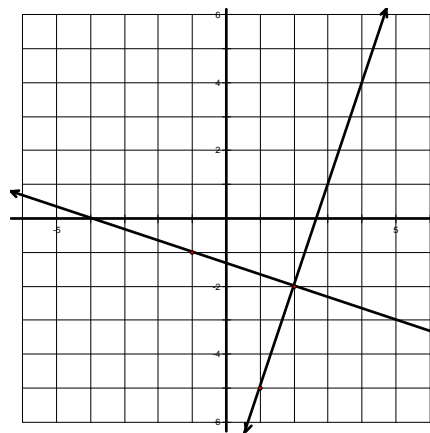
$m = \frac{\text{rise}}{\text{run}} = \frac{\quad}{\quad} = \quad$



If 2 lines are parallel then their slopes are _____

○ **Slopes of perpendicular lines**

The graph shows a pair of perpendicular lines. Calculate the slope of each line in the pair.



$$m = \frac{\text{rise}}{\text{run}} = \frac{\quad}{\quad} = \underline{\quad}$$

$$m = \frac{\text{rise}}{\text{run}} = \frac{\quad}{\quad} = \underline{\quad}$$

Use a protractor to verify that the lines are perpendicular.

If 2 lines are perpendicular then their slopes are _____

● **Section 5**

○ Slope intercept form of a line: _____

○ Standard form of a line: _____

You need to rearrange the equation into standard form, when the problem says to. Otherwise, you can leave the equation in slope intercept form.

<p>Find an equation for a line that is parallel to $y = \frac{3}{4}x + 6$</p> <p>The slope of the line you are looking for is ____.</p> <p>The y intercept can be any number you want.</p> <p>$y = \underline{\quad}x + \underline{\quad}$</p>	<p>Find an equation for a line that is perpendicular to $y = \frac{2}{3}x - 2$</p> <p>The slope of the line you are looking for is ____.</p> <p>The y intercept can be any number you want.</p> <p>$y = \underline{\quad}x + \underline{\quad}$</p>
<p>Find an equation for a line that is parallel to $y = -2x + 3$ and passes through the point (2,3).</p> <p>The slope of the line you are looking for is ____.</p> <p>To find the y intercept, use the point given in the following equation and solve for b:</p> $y = m x + b$ $3 = -2(2) + b$ <p>The equation is for the line is: $y = \underline{\quad}x + \underline{\quad}$</p>	<p>Find an equation for a line that is perpendicular to $y = \frac{4}{5}x + 1$ and passes through the point (2,1).</p> <p>The slope of the line you are looking for is ____.</p> <p>To find the y intercept, use the point given in the following equation and solve for b:</p> $y = m x + b$ $1 = \underline{\quad}(2) + b$ <p>The equation is for the line is: $y = \underline{\quad}x + \underline{\quad}$</p>

If the equation that you are given is in standard form and you need the slope of that line, you can solve the equation for the variable y or you can find 2 points on the line and find the slope between those 2 points.

To find 2 points on the line (without graphing), substitute 0 for x and then substitute 0 for y.

$$3x + 2y = 6 \rightarrow (0, \underline{\quad}) \quad 3(0) + 2y = 6 \quad \rightarrow (\underline{\quad}, 0) \quad 3x + 2(0) = 6$$

$$\quad \quad \quad 0 + 2y = 6 \quad \quad \quad \quad \quad \quad \quad 3x + 0 = 6$$

$$\quad \quad \quad 2y = 6 \quad \quad \quad \quad \quad \quad \quad 3x = 6$$

$$\quad \quad \quad y = 3 \quad \quad \quad \quad \quad \quad \quad x = 2$$

(0,3) (2,0)

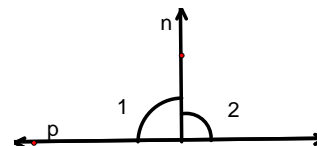
Then you can find the slope between these points.

• **Section 6**

- Theorems about perpendicular lines

- **If 2 lines form a pair of congruent adjacent angles, then they are _____.**

Given: $\angle 1 \cong \angle 2$

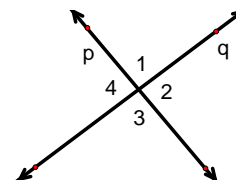


Prove: line $n \perp$ line p

$m\angle \underline{\quad} + m\angle \underline{\quad} = \underline{\quad}^\circ$	linear pair
$m\angle \underline{\quad} + m\angle 2 = \underline{\quad}^\circ$	given
$2m\angle 2 = \underline{\quad}^\circ$	substitution
$m\angle 2 = \underline{\quad}^\circ$	algebra
	division property

- **If 2 lines are perpendicular, then they form ___ right angles.**

Given: line $p \perp$ line q

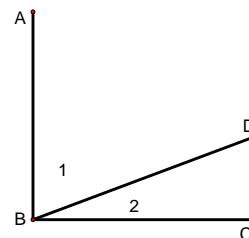


Prove: $m\angle 1 = m\angle 2 = m\angle 3 = m\angle 4 = 90^\circ$

$m\angle 1 = 90^\circ$	definition of \perp
$m\angle 1 + m\angle 2 = 180^\circ$	linear pair
$\underline{\quad} + m\angle 2 = 180^\circ$	
$m\angle 2 = \underline{\quad}^\circ$	
similarly, $\angle 3$ and $\angle 4$ are also 90°	

- **If 2 sides of adjacent acute angles are perpendicular, then the angles are _____.**

Given: $\overline{AB} \perp \overline{BC}$



Prove: $m\angle 1 + m\angle 2 = 90^\circ$

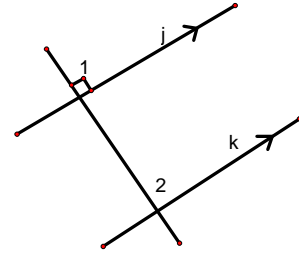
$\overline{AB} \perp \overline{BC}$	
$m\angle ABC = 90^\circ$	def of _____
$m\angle 1 + m\angle 2 = m\angle ABC$	_____ postulate
	substitution

- **If a transversal is perpendicular to one of 2 parallel lines, then it is _____ to the other line also.**

Given: $\angle 1$ is a right angle
line $j \parallel$ line k

Prove: $\angle 2$ is a right angle

line ____ \parallel line ____ _____
 $\angle 1$ is a right angle _____
 $m\angle 1 = 90^\circ$ _____
 $m\angle 1 = m\angle 2$ _____
 _____ = $m\angle 2$ _____

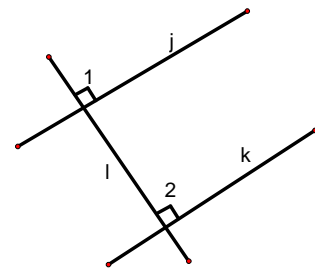


- **In a plane, if 2 lines are perpendicular to the same line then the lines are _____ to each other.**

Given: line $j \perp$ line l
line $k \perp$ line l

Prove: line $j \parallel$ line k

_____ $m\angle 1 = 90^\circ$ _____
 _____ $m\angle 2 = 90^\circ$ _____
 $\angle 1 \cong \angle 2$ _____



<p>Find the value of x that makes line $m \parallel$ line n.</p>	<p>Find x</p>
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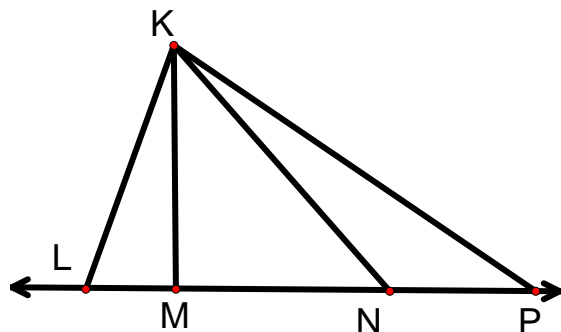
- Distance from a point to a line = _____

Measure (with a ruler) the segments shown.

$KL =$ ____ $KM =$ ____ $KN =$ ____ $KP =$ ____

What do you notice about the length of \overline{KM} ?

What relationship exists between \overline{KM} and \overline{LP} ?



Given: $\overline{BA} \perp \overline{BC}$

Prove: $\angle 1$ and $\angle 2$ are complementary

$\overline{BA} \perp \overline{BC}$

\angle _____ is a right angle

$m\angle ABC =$ _____

$m\angle ABC = m\angle$ _____ $+ m\angle$ _____

$m\angle 1 + m\angle 2 =$ _____^o

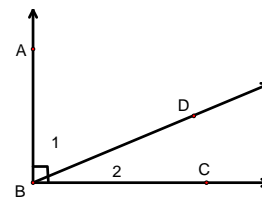
definition of \perp lines

definition of _____

_____ postulate

substitution property

definition of _____



Given: line $j \perp$ line k

$\angle 1$ and $\angle 2$ are a linear pair

Prove: $\angle 2$ is a right angle

_____ + _____ = _____^o

$\angle 1$ is a right angle

$m\angle 1 =$ _____^o

_____ + $m\angle 2 =$ _____^o

$m\angle 2 =$ _____^o

given

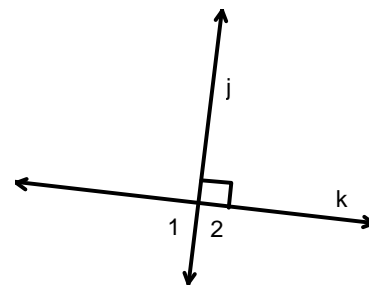
linear pair postulate

given

substitution

subtraction

definition of _____



• Constructions

○ Perpendicular lines

- 1) start with a segment (if you are given a line to work from, just choose 2 points on the line)
- 2) set your compass so that it is a little more than half the length of the segment
- 3) make 2 arcs (one above and one below the segment) from each endpoint
- 4) connect the intersections of the arcs

○ Perpendicular line through a specific point

- 1) using the specified point, make 2 compass arcs on the line
- 2) use these 2 intersections to make arcs above and below the line given
- 3) connect the intersections of the arcs made in step 2

- Square

- 1) make a line to work from
- 2) construct 2 segments that are perpendicular to the first line
- 3) with your compass, measure the distance between the 2 perpendiculars
- 4) use this compass setting to make 2 arcs on the perpendicular segments
- 5) connect between the 2 marks made in step 4

- Parallel lines

- 1) draw the first of the 2 parallel lines
- 2) draw a transversal (just a line that intersects the first line for now)
- 3) make a mark on the transversal where the parallel line will intersect
- 4) with your compass, make 2 arcs from the intersection of the first parallel and the transversal
- 5) use the same compass setting to make 2 arcs from the mark for the 2nd parallel—one on the transversal and the other where you expect the parallel to come through
- 6) use your compass to MEASURE between the arcs from the first parallel intersection
- 7) transfer this measurement to the 2nd parallel
- 8) connect the arc intersection to the point made in step 3