

# Essentials of Geometry

Before you begin:

- There is a lot of vocabulary in chapter 1 on purpose.
  - You need to KNOW these words—do not just memorize them—USE them.
  - Look for connections between the words and their meanings—they will be easier to learn.
- Once you know these words, we can talk and we both know what the other is saying.
- **Section 1**—basic vocabulary

Definition v. description

Definition = \_\_\_\_\_

Description = \_\_\_\_\_

There are some terms that we use every day that are very difficult to define and yet we understand them well. There are 3 such ‘undefined terms’ in geometry.

Undefined Term	Brief Description	Picture	Name/Symbol
Point			
Line			
Plane			

Collinear points = \_\_\_\_\_

Draw a line that passes through points A and B.

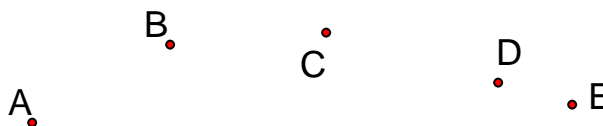
Points A and B are **collinear**.

Draw a line that passes through points C, D and E

Points C, D and E are **collinear**.

Draw a line through points A, B and C

Points A, B and C are **non-collinear**



Coplanar points = \_\_\_\_\_

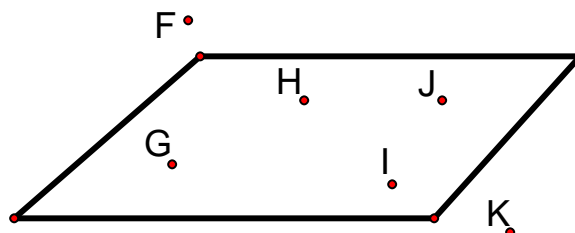
Coplanar lines = \_\_\_\_\_

Name 3 coplanar points: \_\_\_\_, \_\_\_\_, and \_\_\_\_

Name 4 non-coplanar points: \_\_\_\_, \_\_\_\_,  
\_\_\_\_, and \_\_\_\_

Name 2 coplanar lines: \_\_\_\_ and \_\_\_\_

Name 2 non-coplanar lines: \_\_\_\_ and \_\_\_\_



Draw 2 lines that intersect a plane at the same point.	Draw 2 planes that intersect in a line.	Draw 2 planes that do not intersect.
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	Brief description	Picture	Name/Symbol	Number of endpoints	Number of points on it
Segment					
Ray					
Line					

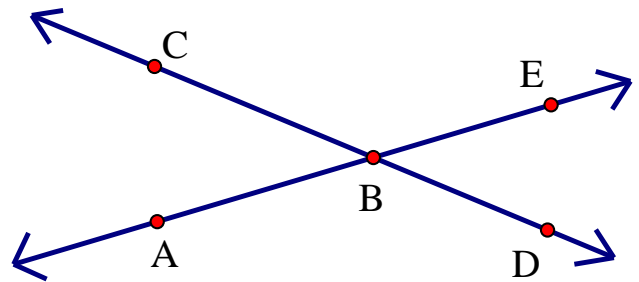
Opposite rays = \_\_\_\_\_

Intersection = \_\_\_\_\_

Name 2 opposite rays: \_\_\_\_ and \_\_\_\_

Name 2 intersecting lines:

Name the point of intersection of the 2 lines: \_\_\_\_\_



Are the rays  $\overrightarrow{AB}$  and  $\overrightarrow{BA}$  opposite rays?  
Explain.

Draw lines $k$ and $l$ that intersect at point $Z$	Draw lines $m$ and $n$ in such a way that they do not intersect
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Draw  $\overleftrightarrow{AB}$  intersecting  $\overleftrightarrow{BC}$

Draw  $\overleftrightarrow{AB}$  intersecting  $\overleftrightarrow{CD}$

Draw 4 non-collinear points—  
name them A, B, C and D.

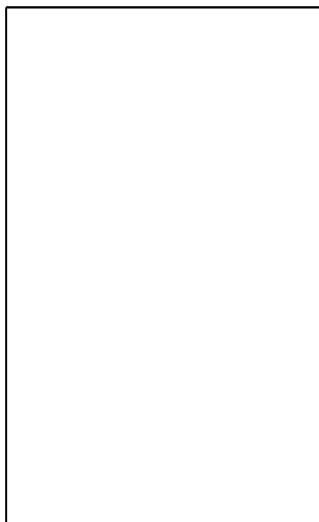
Draw  $\overleftrightarrow{AB}$

Draw  $\overleftrightarrow{BC}$

Draw  $\overleftrightarrow{CD}$

Draw  $\overleftrightarrow{DA}$

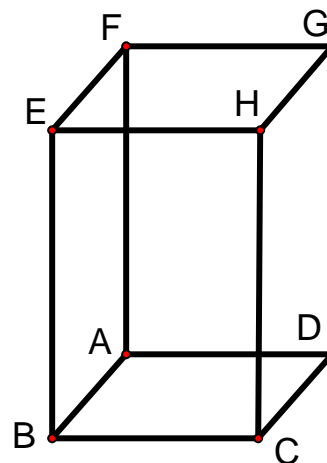
Draw  $\overleftrightarrow{BD}$



Name 3 points in the diagram  
that are not collinear.

Name a point that is coplanar  
with A, B and H.

Name 4 non-coplanar points.



### SKILL PRACTICE

1. Vocabulary—Write what each of the following symbols means.
  - a. Q
  - b.  $\overline{MN}$
  - c.  $\overrightarrow{ST}$
  - d.  $\overleftrightarrow{FG}$
  
2. Writing—Compare collinear points and coplanar points. Are collinear points also coplanar? Are coplanar points also collinear? Explain.

NAMING POINTS, LINES, and PLANES Use the diagram for exercises 3-7.

3. Give 2 other names for  $\overleftrightarrow{WQ}$ .

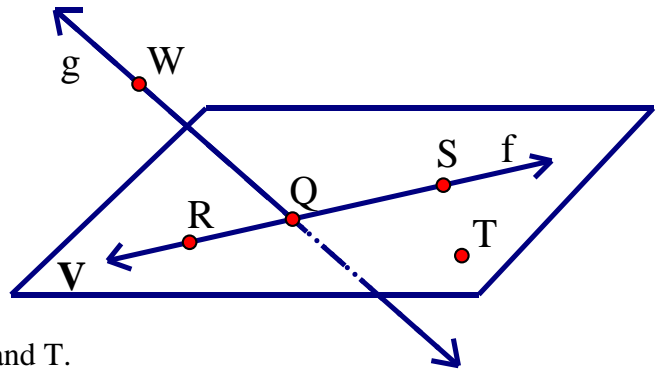
4. Give another name for plane V.

5. Name 3 points that are collinear.

Name a 4<sup>th</sup> point that is NOT collinear with the other 3.

6. Name a point that is NOT coplanar with R, S, and T.

7. WRITING—Is point W coplanar with points Q and R? Explain.



NAMING SEGMENTS AND RAYS Use the diagram for exercises 8-12.

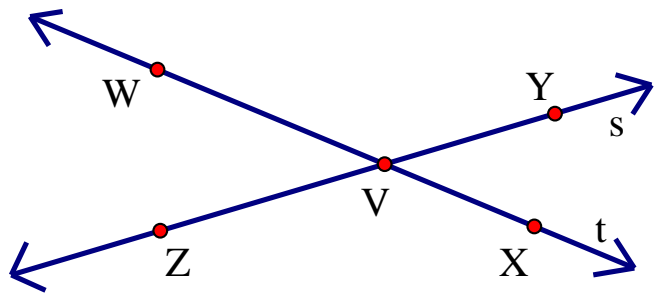
8. What is another name for  $\overleftrightarrow{ZY}$ ?

9. Name all rays with endpoint V.

10. Name 2 pairs of opposite rays.

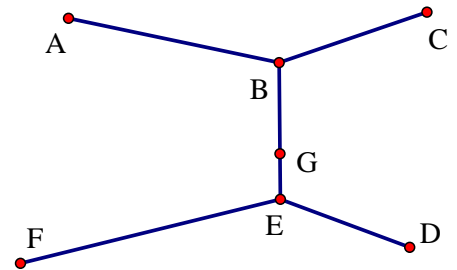
11. Give another name for  $\overleftrightarrow{WV}$ .

12. ERROR ANALYSIS—A student says that  $\overleftrightarrow{VW}$  and  $\overleftrightarrow{VZ}$  are opposite rays because they have the same endpoint. DESCRIBE the error.



13. Which statement about the diagram at the right is true?

- A, B, and C are collinear.
- C, D, E, and G are coplanar.
- B lies on  $\overleftrightarrow{GE}$ .
- $\overleftrightarrow{EF}$  and  $\overleftrightarrow{ED}$  are opposite rays.



SKETCHING INTERSECTIONS Sketch the figure described.

14. Three lines that lie in a plane and intersect at one point.

15. One line that lies in a plane, and one line that does not lie in that plane.

16. Line AB and line CD intersect at point E. Which of the following are opposite rays?

- a.  $\overrightarrow{EC}$  and  $\overrightarrow{ED}$
- b.  $\overrightarrow{CE}$  and  $\overrightarrow{DE}$
- c.  $\overrightarrow{AB}$  and  $\overrightarrow{BA}$
- d.  $\overrightarrow{AE}$  and  $\overrightarrow{BE}$

READING DIAGRAMS In exercises 17-22, use the diagram at the right.

17. Name the intersection of  $\overrightarrow{PR}$  and  $\overrightarrow{HR}$ .

18. Name the intersection of plane EFG and plane FGS.

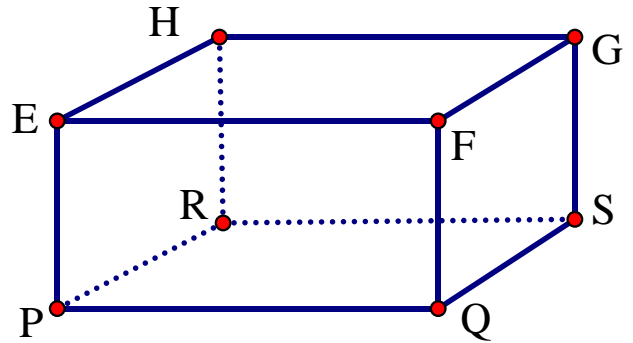
19. Name the intersection of plane PQS and plane HGS.

20. Are points P, Q, and F collinear? Are they coplanar?

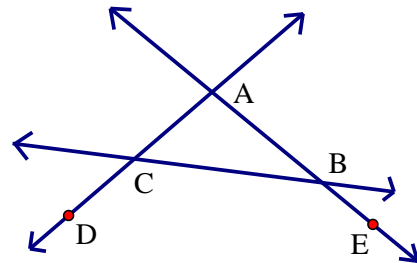
21. Are points P and G collinear? Are they coplanar?

22. Name 3 planes that intersect at point E.

23. SKETCHING PLANES—Sketch plane J intersecting plane K. Then draw a line  $l$  in plane J that intersects plane K at a single point.



24. NAMING RAYS—Name 10 different rays in the diagram at the right. Then name 2 pair of opposite rays.



25. SKETCHING—Draw 3 noncollinear points J, K, and L. Sketch  $\overline{JK}$  and add a point M on  $\overline{JK}$ . Then sketch  $\overline{ML}$ .

26. SKETCHING—Draw 2 points P and Q. Then sketch  $\overrightarrow{PQ}$ . Add a point R on the ray so that Q is between P and R.

ALGEBRA In exercises 27-32, you are given an equation of a line and a point. Use substitution to determine whether the point is on the line.

27.  $y = x - 4$ ; A (5, 1)

28.  $y = x + 1$ ; A (1, 0)

29.  $y = 3x + 4$ ; A (7, 1)

30.  $y = 4x + 2$ ; A (1, 6)

31.  $y = 3x - 2$ ; A (-1, -5)

32.  $y = -2x + 8$ ; A (-4, 0)

GRAPHING Graph the inequality on a number line. Tell whether the graph is a *segment*, a *ray* or *rays*, a *point*, or a *line*.

33.  $x \leq 3$

34.  $x \geq -4$

35.  $-7 \leq x \leq 4$

36.  $x \geq 5$  or  $x \leq -2$

37.  $x \geq -1$  or  $x \leq 5$

38.  $|x| \leq 0$

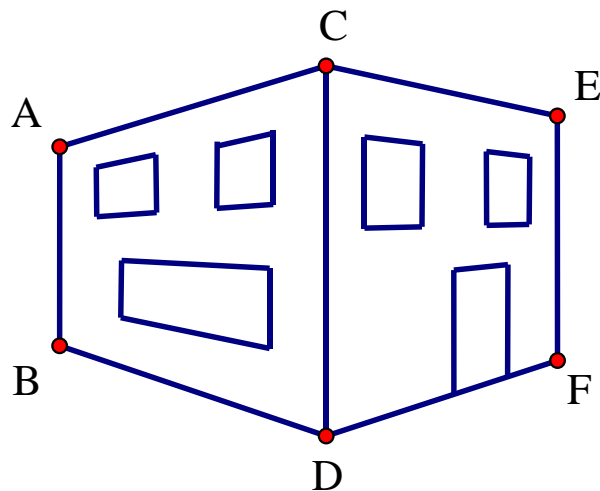
39. CHALLENGE—Tell whether each of the following statements involving 3 planes is possible. If a situation is possible, make a sketch.

- a. None of the 3 planes intersect.
- b. The 3 planes intersect in one line.
- c. The 3 planes intersect in one point.
- d. 2 planes do not intersect. The 3<sup>rd</sup> plane intersects the other 2.
- e. Exactly 2 planes intersect. The 3<sup>rd</sup> plane does not intersect the other 2.

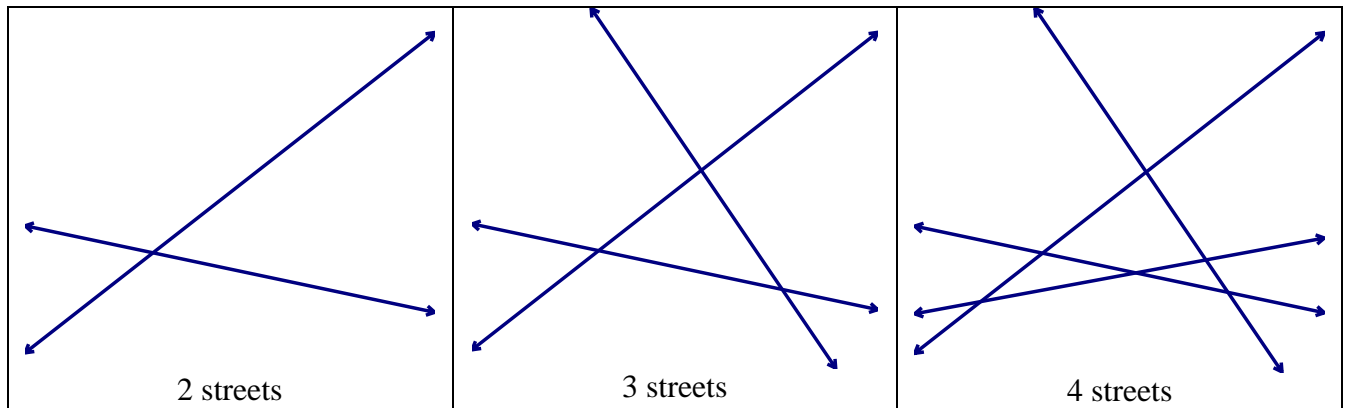
### PROBLEM SOLVING

(problems 40-42 involved photographs)

43. **SHORT RESPONSE**—Explain why a 4-legged table may rock from side to side even if the floor is level. Would a 3-legged table on the same level floor rock from side to side? Why or why not?
44. **SURVEYING**—A surveying instrument is placed on a tripod. The tripod has 3 legs whose lengths can be adjusted.
- When the tripod is sitting on a level surface, are the tips of the legs coplanar?
  - Suppose the tripod is used on a sloping surface. The length of each leg is adjusted so that the base of the surveying instrument is level with the horizon. Are the tips of the legs coplanar? Explain.
45. **MULTI-STEP PROBLEM**—In a *perspective drawing*, lines that do not intersect in real life are represented by lines that appear to intersect at a point far away on the horizon. This point is called a *vanishing point*. The diagram shows a drawing of a house with 2 vanishing points.
- Extend  $\overline{AC}$  to the left—use a straight edge so that the drawing is neat. Extend  $\overline{BD}$  also to the left. Label the point where these intersect  $V$ .  $V$  is the first vanishing point. Find the second vanishing point by extending  $\overline{CE}$  and  $\overline{DF}$  to the right. Label this vanishing point  $W$ .
  - Draw  $\overline{AW}$  and  $\overline{EV}$ . Label their intersection point  $G$ . Label the intersection of  $\overline{FV}$  and  $\overline{BW}$  as  $H$ .
  - Using heavy dashed lines, draw the hidden edges of the house:  $\overline{AG}$ ,  $\overline{EG}$ ,  $\overline{BH}$ ,  $\overline{FH}$ , and  $\overline{GH}$ .



46. CHALLENGE—Each street in a particular town intersects every existing street exactly one time. Only 2 streets pass through each intersection.



- A traffic light is needed at each intersection. How many traffic lights are needed if there are 5 streets in town? 6 streets?
- Describe* a pattern you can use to find the number of additional traffic lights that are needed each time a street is added in the town.

**Section 2**—segments and congruence

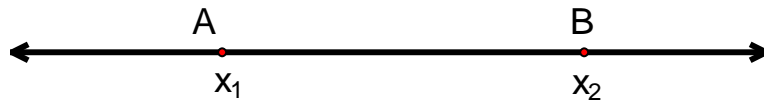
Postulate (a.k.a. axiom) = \_\_\_\_\_

All mathematical rules are ALWAYS true—no exceptions.  
Keep the number of postulates to a minimum.

Coordinate = \_\_\_\_\_

Distance = \_\_\_\_\_

**RULER POSTULATE**

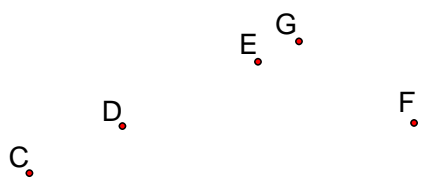


- The capital letters A and B are the names of the points. The lower case letters with the subscripts refer to the locations of the points. The locations are numbers (like from a number line). The distance between the points is the absolute value of the difference of these numbers-- $|x_2 - x_1|$

- $\overleftrightarrow{AB}$  = \_\_\_\_\_
- $\overline{AB}$  = \_\_\_\_\_
- $AB$  = \_\_\_\_\_

Between = \_\_\_\_\_

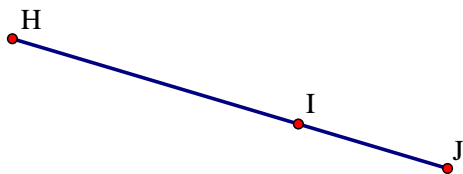
(Try to give a 'good' definition without using the word 'between' anywhere in your definition.)



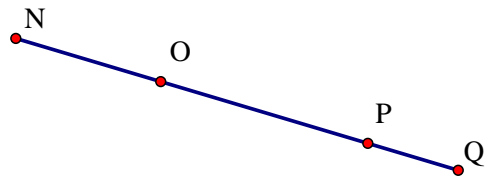
Point E is between G and D.  
G is NOT between E and F because \_\_\_\_\_.  
D and E are both between C and G.

The points MUST be collinear in order for one point to be between the other two.

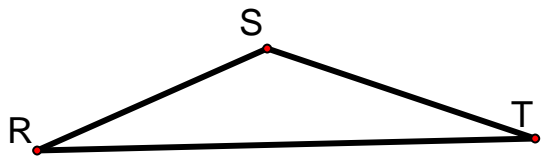
**SEGMENT ADDITION POSTULATE**



$HI + IJ =$  \_\_\_\_\_  
 $HJ - IJ =$  \_\_\_\_\_



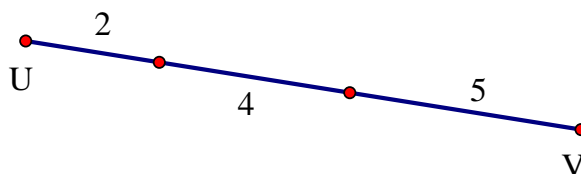
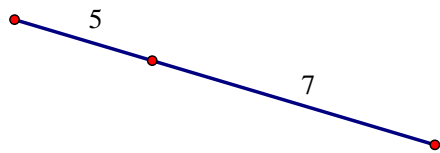
$NO + OP =$  \_\_\_\_\_       $OQ + ON =$  \_\_\_\_\_  
 $NO + OP + PQ =$  \_\_\_\_\_       $QO - OP =$  \_\_\_\_\_



The segment addition postulate only works if the points are collinear.

$$TR \neq \underline{\quad} + \underline{\quad}$$

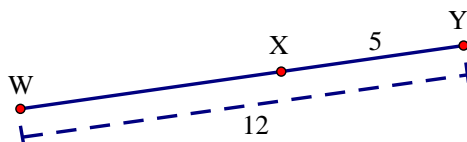
TR is shorter than the sum of SR and ST because \_\_\_\_\_.



The whole segment measures \_\_\_\_\_

$$UV = \underline{\quad}$$

$$WX = \underline{\quad}$$



Congruent segments = \_\_\_\_\_

- The symbol for 'congruent' is \_\_\_\_\_.  
This symbol is used when you are writing the NAMES of the objects.
- The symbol for 'equal' is \_\_\_\_\_.  
This symbol is used when you are writing NUMBERS (such as the lengths of the segments).

You will get comfortable with the symbol for congruent since you will see it a lot. For now, think of it as pretty much the same as an equal sign.

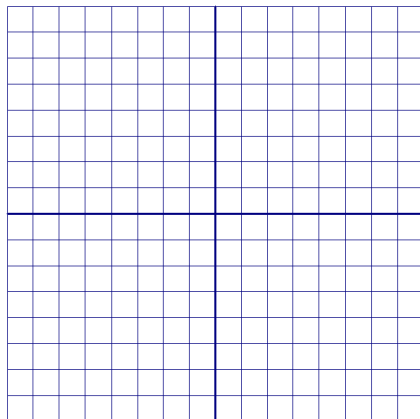
Plot the points:

- A (-2,3)    B (3,3)  
C (-3,4)    D (-3,-1)

AB = \_\_\_\_\_ units  
CD = \_\_\_\_\_ units

AB \_\_\_\_\_ CD (= or  $\neq$ )

Are the segments congruent?



Plot the points:

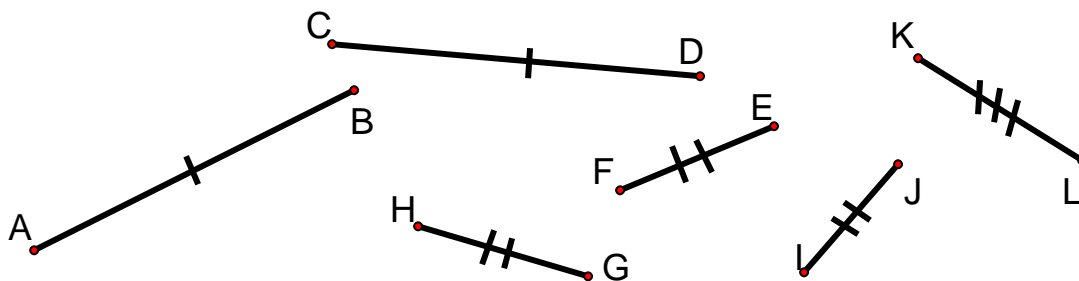
- E (0,5)    F (0,-1)  
G (4,0)    H (-1,0)

EF = \_\_\_\_\_ units  
GH = \_\_\_\_\_ units

EF \_\_\_\_\_ GH (= or  $\neq$ )

Are the segments congruent?

When 2 (or more) segments are the same length, you will know because the picture will be marked.



In the illustration above,  $\overline{AB}$  and  $\overline{CD}$  are congruent since they have the same number of marks on them. Segments  $\overline{EF}$ ,  $\underline{\hspace{1cm}}$  and  $\underline{\hspace{1cm}}$  are congruent. Segment  $\underline{\hspace{1cm}}$  is a different length from  $\overline{AB}$  and  $\overline{EF}$ .

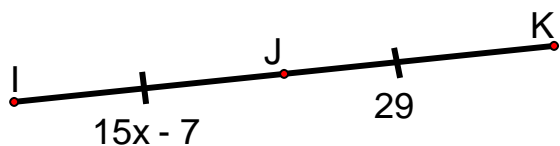
- Any segments that have the same number of marks are the same length.
- If they have a different number of marks, they are different lengths.
- You will not normally see more than 3 ticks on the same segment.

Setting up and Solving equations in geometry.

Sometimes you will be asked to find the value of the variable.

Sometimes you will be asked to find the length of the segment.

Be sure you know what you are being asked to find.



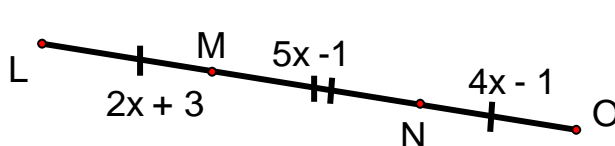
Find 2 segments that are the same length.  
Write an equation that sets them equal.

$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

Solve the equation for the variable.

$x = \underline{\hspace{1cm}}$      $IJ = \underline{\hspace{1cm}}$      $JK = \underline{\hspace{1cm}}$

$IK = \underline{\hspace{1cm}}$



Find 2 segments that are the same length.  
Write an equation that sets them equal.

$\underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

Solve the equation for the variable.

$x = \underline{\hspace{1cm}}$      $LM = \underline{\hspace{1cm}}$      $NO = \underline{\hspace{1cm}}$

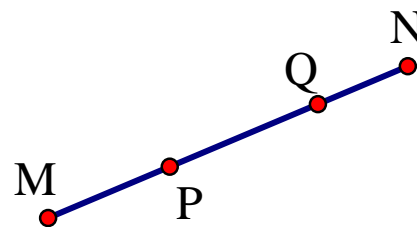
$MN = \underline{\hspace{1cm}}$

$MO = \underline{\hspace{1cm}}$

$LO = \underline{\hspace{1cm}}$

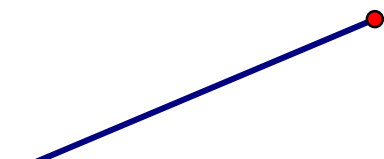



**SKILL PRACTICE**

- VOCABULARY—*Explain* what  $\overline{MN}$  means and what MN means.
- WRITING—*Explain* how you can find PN if you know PQ and QN.



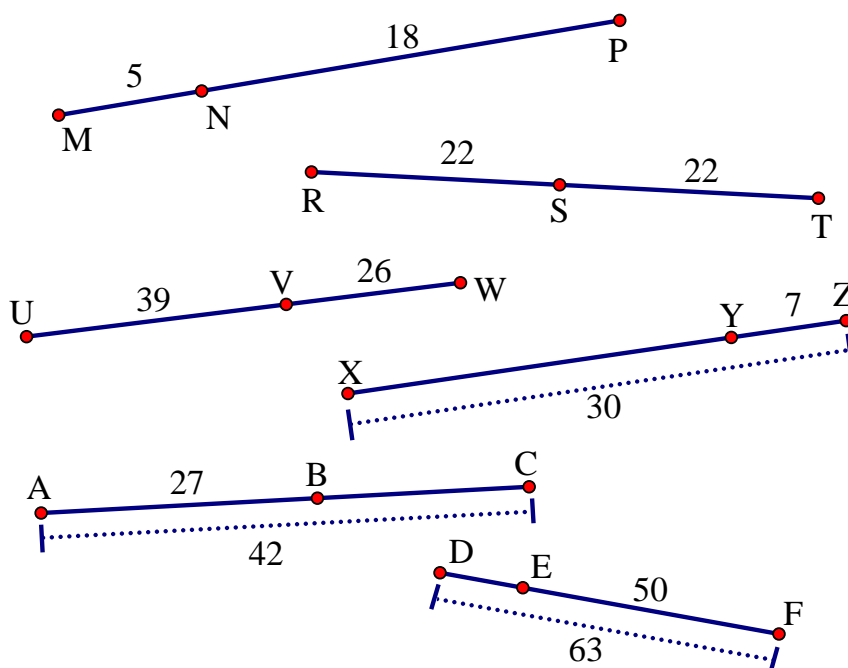
How can you find PN if you know MP and MN?

MEASUREMENT Measure the length of each segment to the nearest tenth of a cm.

- 
- 
- 
- 

SEGMENT ADDITION POSTULATE Find the indicated length.

- Find MP.  
(Do not use a ruler.)
- Find RT.
- Find UW.
- Find XY.
- Find BC.
- Find DE.



12. ERROR ANALYSIS—In the figure at the right,  $AC = 14$  and  $AB = 9$ . Describe and correct the error made in finding  $BC$ .



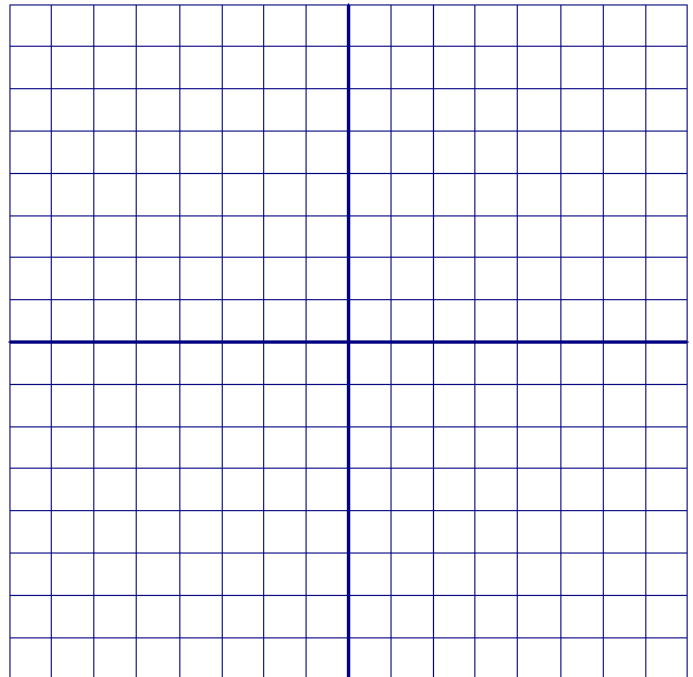
$$BC = 14 + 9 = 23$$

CONGRUENCE In exercises 13-15, plot the given points in a coordinate plane. Then determine whether the line segments named are congruent.

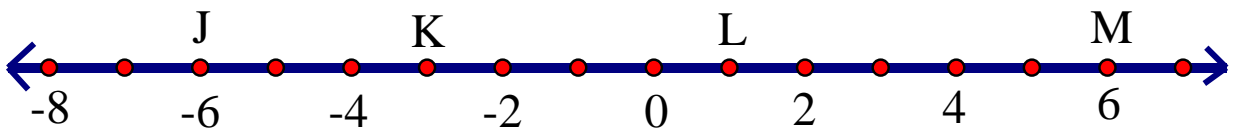
13.  $A(0, 1)$ ,  $B(4, 1)$ ,  $C(1, 2)$ ,  $D(1, 6)$ ;  
 $\overline{AB}$  and  $\overline{CD}$

14.  $J(-6, -8)$ ,  $K(-6, -2)$ ,  $L(-2, -4)$ ,  
 $M(-6, -4)$ ;  $\overline{JK}$  and  $\overline{LM}$

15.  $R(-200, 300)$ ,  $S(200, 300)$ ,  
 $T(300, -200)$ ,  $U(300, 100)$ ;  
 $\overline{RS}$  and  $\overline{TU}$   
 (You can use creative graphing for this one.)



ALGEBRA Use the number line to find the indicated distance.



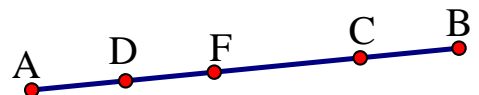
16.  $JK = \underline{\hspace{2cm}}$

17.  $JL = \underline{\hspace{2cm}}$

18.  $JM = \underline{\hspace{2cm}}$

19.  $KM = \underline{\hspace{2cm}}$

20. SHORT RESPONSE—Use the diagram. Is it possible to use the Segment Addition Postulate to show that  $FB > CB$  or that  $AC > DB$ ? Explain.



**FINDING LENGTHS** In the diagram, points V, W, X, Y, and Z are collinear.  $VZ = 52$ ,  $XZ = 20$  and  $WX = XY = YZ$ . Find the indicated length.

21. WX

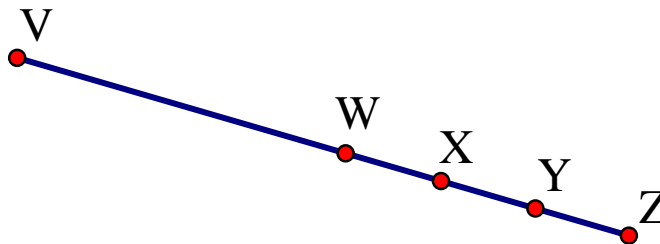
22. VW

23. WY

24. VX

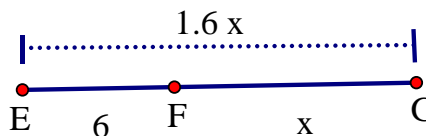
25. WZ

26. VY



27. Use the diagram. What is the length of  $\overline{EG}$ ?

- a. 1
- b. 4.4
- c. 10
- d. 16



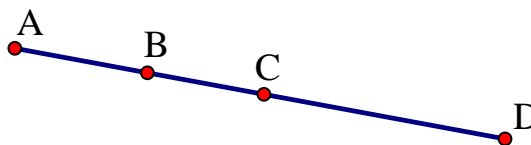
**ALGEBRA** Point S is between R and T on  $\overline{RT}$ . Use the given information to write an equation in terms of x. Solve the equation. Then find RS and ST.

28.  $RS = 2x + 10$   
 $ST = x - 4$   
 $RT = 21$

29.  $RS = 3x - 16$   
 $ST = 4x - 8$   
 $RT = 60$

30.  $RS = 2x - 8$   
 $ST = 3x - 10$   
 $RT = 17$

31. **CHALLENGE**—In the diagram,  $\overline{AB} \cong \overline{BC}$ ,  $\overline{AC} \cong \overline{CD}$ , and  $AD = 12$ . Find the lengths of all the segments in the diagram. Suppose you choose one of the segments at random. What is the probability that the measurement of the segment is greater than 3? Explain.



**PROBLEM SOLVING**

(32-34 involve photographs)

35. **MULTI-STEP PROBLEM**—A climber used a rope to descend a vertical cliff. Let A represent the point where the rope is secured at the top of the cliff. Let B represent the climber’s position. Let C represent the point where the rope is secured at the bottom of the cliff.

a. **MODEL**—Draw and label a line segment that represents this situation.

b. **CALCULATE**—If AC is 52 feet and AB is 31 feet, how much farther must the climber descend to reach the bottom of the cliff?

36. **CHALLENGE**—Four cities lie along a straight highway in this order: City A, City B, City C, and City D. The distance from City A to City B is 5 times the distance from City B to City C. The distance from City A to City D is 2 times the distance from City A to City B. Complete the mileage chart.

	City A	City B	City C	City D
City A	XXXXXXXXXX			
City B		XXXXXXXXXX		
City C			XXXXXXXXXX	10 miles
City D				XXXXXXXXXX

**Section 3**—midpoint and distance formulas

Midpoint = \_\_\_\_\_

The midpoint is a POINT. It is a precise location.

Segment bisector = \_\_\_\_\_

**A bisector is a thing which makes the word ‘bisector’ a NOUN. This thing may or may not be a point. It could be a line, a ray, a segment, a plane, or even a point.**

Bisect = \_\_\_\_\_

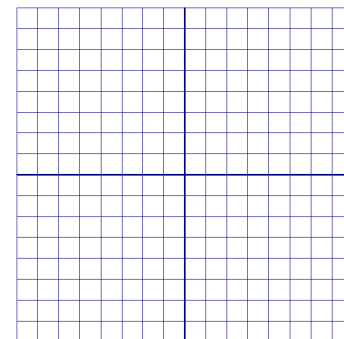
To bisect something is an action which makes the word ‘bisect’ a VERB.

MIDPOINT FORMULA = \_\_\_\_\_

- Find the location of the midpoint between the points (6,3) and (-2,4) by using the formula.

Plot the points to see if your calculation looks correct.

- Find the location of the midpoint between the points (-1,-4) and (-6,5) by using the formula.



DISTANCE FORMULA = \_\_\_\_\_

This formula is derived from the Pythagorean Theorem which looks like: \_\_\_\_\_

- Plot the points A (2,4) and B (5,-1) on the graph.
- The distance between these points (or the length of the segment) cannot simply be ‘counted’ off the graph paper.
- Draw a segment straight down from point A.
- Draw a segment straight across from B.
- These 2 segments intersect at point C.

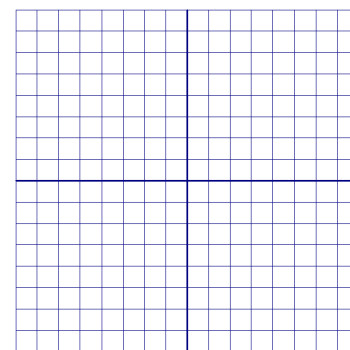
AC = \_\_\_\_ BC = \_\_\_\_ The lengths of these segments can be counted off the grid since they do not ‘slant’.

- Use the lengths of these (AC and BC) segments as the values of ‘a’ and ‘b’ in the Pythagorean Theorem. Solve for ‘c’ which is the distance from point A to B.

- Now calculate the distance from A to B using the distance formula.

$$AB = \sqrt{(\_\_ - \_\_)^2 + (\_\_ - \_\_)^2}$$

**Both of these calculations should give the exact same answer.**



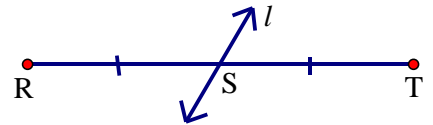
- Find the distance between the points (2,3) and (4,-1) using the formula.
- Find the distance between the points (-1,2) and (3,-2) using the formula.

**SKILL PRACTICE**

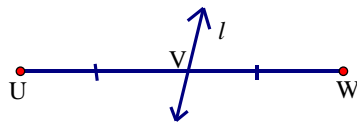
1. VOCABULARY—Fill in the blank: To find the length of  $\overline{AB}$ , with endpoints at A (-7, 5) and B (4, -6), you can use the \_\_\_\_\_.
2. WRITING—*Explain* what it means to bisect a segment. Why is it impossible to bisect a line?

FINDING LENGTHS Line  $l$  bisects the segment. Find the indicated length.

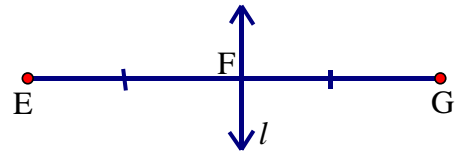
3. Find RT if RS = 5 1/8 inch



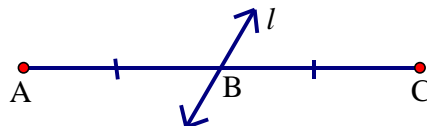
4. Find UW if VW = 5/8 inch



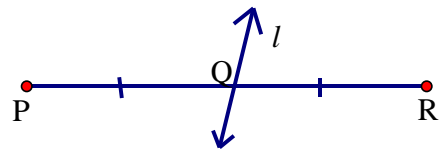
5. Find EG if EF = 13 cm



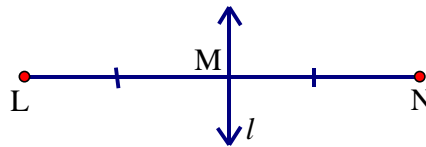
6. Find BC if AC = 19 cm



7. Find QR if PR = 9 1/2 inch



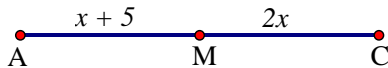
8. Find LM if LN = 137 mm



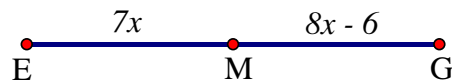
9. SEGMENT BISECTOR—Line RS bisects  $\overline{PQ}$  at point R. Find RQ if PQ 4 3/4 inches.
10. SEGMENT BISECTOR—Point T bisects  $\overline{UV}$ . Find UV if UT = 2 7/8 inches.

ALGEBRA In each diagram, M is the midpoint of the segment. Find the indicated length.

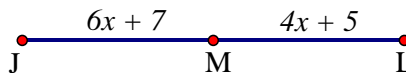
11. Find AM.



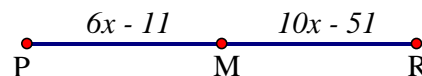
12. Find EM.



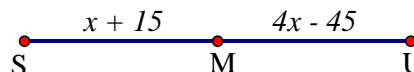
13. Find JM.



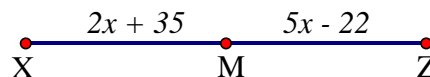
14. Find PR.



15. Find SU.



16. Find XZ.



FINDING MIDPOINTS Find the coordinates of the midpoint of the segment with the given endpoints.

17. C (3, 5) and D (7, 5)

18. E (0, 4) and F (4, 3)

19. G (-4, 4) and H (6, 4)

20. J (-7, -5) and K (-3, 7)

21. P (-8, -7) and Q (11, 5)

22. S (-3, 3) and T (-8, 6)

23. WRITING—Develop a formula for finding the midpoint of a segment with endpoints A (0, 0) and B (m, n). *Explain* your thinking.

24. ERROR ANALYSIS—Describe the error made in finding the coordinates of the midpoint of a segment with endpoints S (8, 3) and T (2, -1).  $\left(\frac{8-2}{2}, \frac{3-(-1)}{2}\right) = (3, 2)$

FINDING ENDPOINTS Use the given endpoint R and midpoint M of  $\overline{RS}$  to find the coordinates of the other endpoint S.

25. R (3, 0), M (0, 5)

26. R (5, 1), M (1, 4)

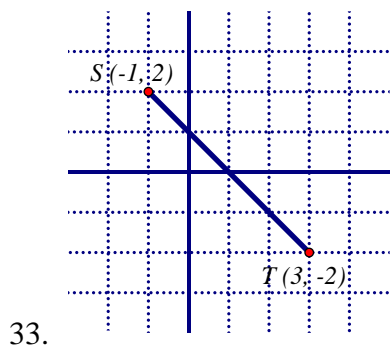
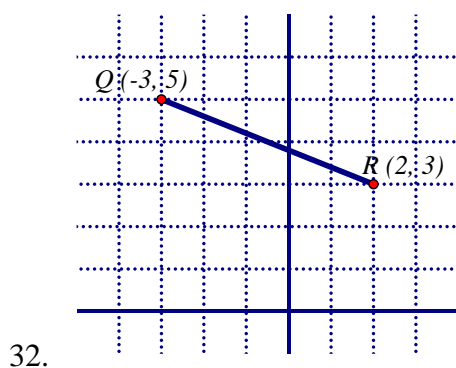
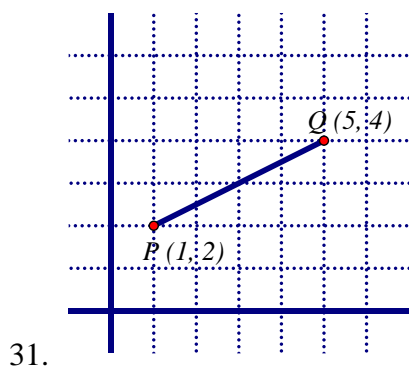
27. R (6, -2), M (5, 3)

28. R (-7, 11), M (2, 1)

29. R (4, -6), M (-7, 8)

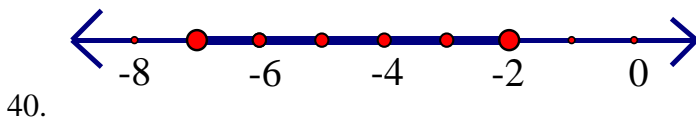
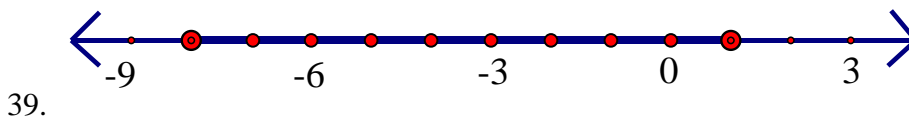
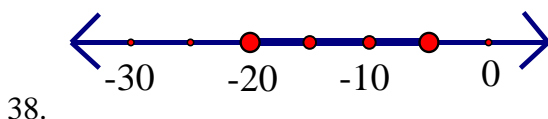
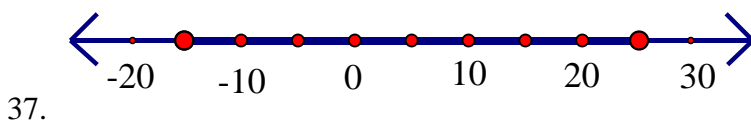
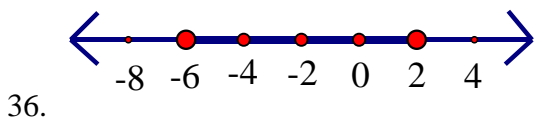
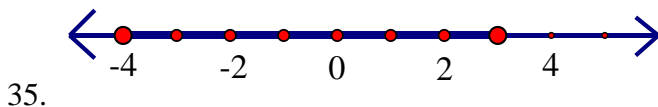
30. R (-4, -6), M (3, -4)

DISTANCE FORMULA Find the length of the segment. Round to the nearest tenth of a unit.



34. The endpoints of  $\overline{MN}$  are M (-3, -9) and N (4, 8). What is the approximate length of  $\overline{MN}$ ?
- 1.4 units
  - 7.2 units
  - 13 units
  - 18.4 units

NUMBER LINE Find the length of the segment. Then find the coordinate of the midpoint of the segment.



41. The endpoints of  $\overline{LF}$  are L (-2, 2) and F (3, 1). The endpoints of  $\overline{JR}$  are J (1, -1) and R (2, -3). What is the approximate difference in the lengths of the 2 segments?
- 2.24
  - 2.86
  - 5.10
  - 7.96

42. SHORT RESPONSE—One endpoint of  $\overline{PQ}$  is P (-2, 4). The midpoint of  $\overline{PQ}$  is M (1, 0).  
*Explain how to find PQ.*

COMPARING LENGTHS The endpoints of 2 segments are given. Find each segment length. Tell whether the segments are congruent.

43.  $\overline{AB}$ : A (0, 2), B (-3, 8)       $\overline{CD}$ : C (-2, 2), D (0, 4)

44.  $\overline{EF}$ : E (1, 4), F (5, 1)       $\overline{GH}$ : G (-3, 1), H (1, 6)

45.  $\overline{JK}$ : J (-4, 0), K (4, 8)       $\overline{LM}$ : L (-4, 2), M (3, -7)

46. ALGEBRA—Points S, T, and P lie on a number line. Their coordinates are 0, 1, and x respectively. Given  $SP = PT$ , what is the value of x?

47. CHALLENGE—M is the midpoint of  $\overline{JK}$ ,  $JM = \frac{x}{8}$ , and  $JK = \frac{3x}{4} - 6$ . Find MK.

**PROBLEM SOLVING**

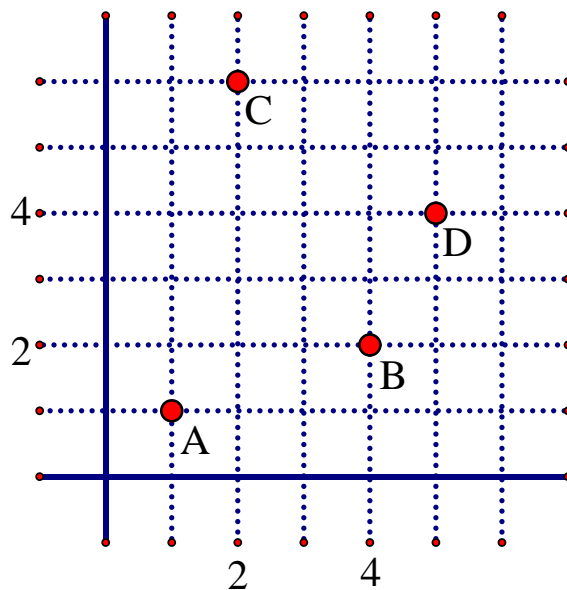
(problem 48 involves a photograph)

49. DISTANCES—A house and a school are 5.7 km apart on the same straight road. The library is on the same road, halfway between the house and the school. Draw a sketch to represent this situation. Mark the locations of the house, school, and library. How far is the library from the house?

ARCHAEOLOGY—The points on the diagram show the positions of objects at an underwater archaeological site. Use the diagram for exercises 50-51.

50. Find the distance between each pair of objects. Round to the nearest tenth of a meter if necessary.

- a. A and B
- b. B and C
- c. C and D
- d. A and D
- e. B and D
- f. A and C

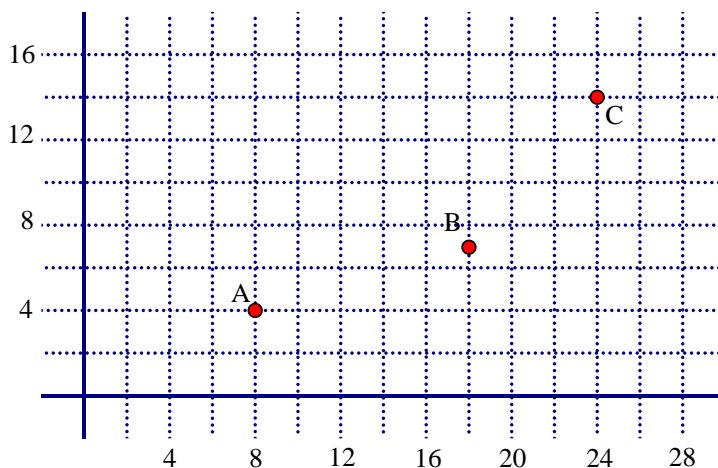


51. Which 2 objects are the closest to each other?

Which are the farthest apart?

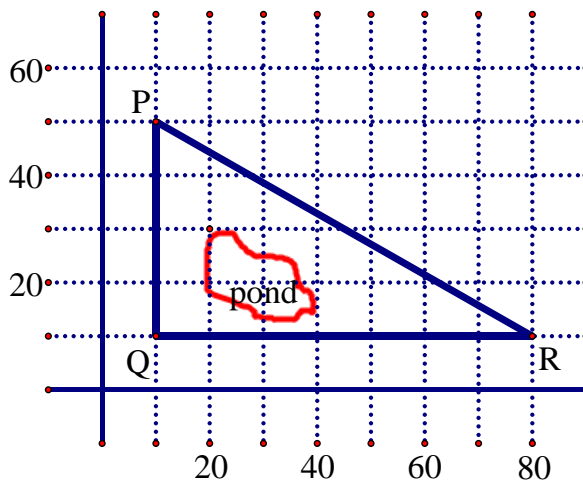
52. WATER POLO—The diagram shows the positions of 3 players during part of a water polo match. Player A throws the ball to Player B, who then throws it to Player C. Round all answers to the nearest tenth of a meter.

- How far did Player A throw the ball?
- How far did Player B throw the ball?
- How far would Player A have thrown the ball if he had thrown it directly to Player C?



53. EXTENDED RESPONSE—As shown, a path goes around a triangular park.

- Find the distance around the park to the nearest yard.
- A new path and a bridge are constructed from point Q to the midpoint M of  $\overline{PR}$ . Find QM to the nearest yard.
- A man jogs from P to Q to M to R to Q and back to P at an average speed of 150 yards per minute. About how many minutes does it take? Explain.



54. CHALLENGE--  $\overline{AB}$  bisects  $\overline{CD}$  at point M,  $\overline{CD}$  bisects  $\overline{AB}$  at M, and  $AB = 4(CM)$ . Describe the relationship between AM and CD.

QUIZ

name \_\_\_\_\_

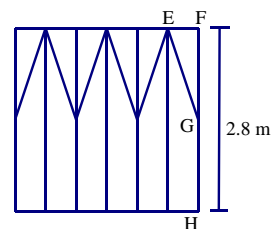
- Sketch 2 lines that intersect the same plane at 2 different points. The lines intersect each other at a point not in the plane.

In the diagram of collinear points,  $AE = 26$ ,  $AD = 15$ , and  $AB = BC = CD$ . Find the indicated lengths.

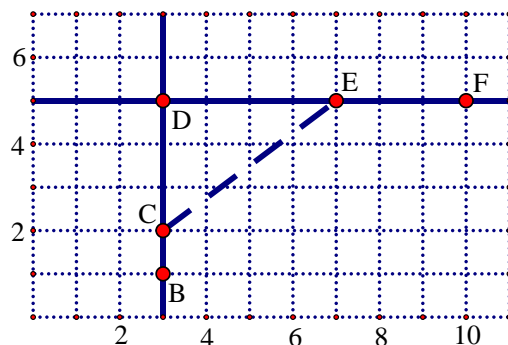
- DE
- AB
- AC
- BD
- CE
- BE



- The endpoints of  $\overline{RS}$  are R (-2, -1) and S (2, 3).
  - Find the coordinates of the midpoint of  $\overline{RS}$ .
  - Find the distance between R and S.
- The diagram shows the frame for a wall.  $\overline{FH}$  represents a vertical board and  $\overline{EG}$  represents a brace. The brace bisects  $\overline{FH}$ . How long is  $\overline{FG}$ ?
  - 0.8 meters
  - 1.4 meters
  - 4.8 meters
  - 5.6 meters
- Point E is the midpoint of  $\overline{AB}$  and  $\overline{CD}$ . The coordinates of A, B, and C are A (-4, 5), B (6, -5), and C (2, 8). What are the coordinates of point D?
  - (0, 2)
  - (1.5, 4)
  - (0, -8)
  - (-10, 2)



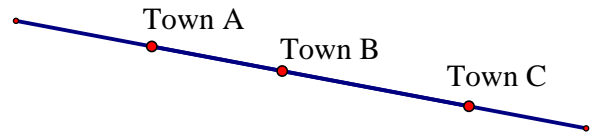
- The diagram shows existing roads and a planned new road, represented by  $\overline{CE}$ . About how much shorter is a trip from B to F, where possible, using the new road instead of the existing roads?
  - 2 miles
  - 5 miles
  - 6 miles
  - 7 miles



12. Rectangle QRST has vertices Q (3, -3), R (0, -5), S (-4, 1), and T (-1, 3). What is the perimeter of rectangle QRST? (Round to the nearest tenth.)
- 7.2 units
  - 14.4 units
  - 21.6 units
  - 28.8 units

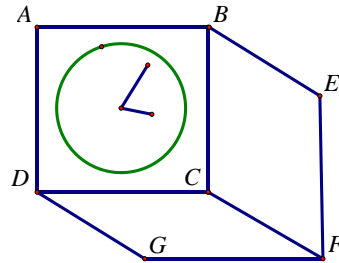
13. Jill is a salesperson who needs to visit Towns A, B, and C. On the map,  $AB = 18.7$  km and  $BC = 2(AB)$ . Starting at Town A, Jill travels along the road shown to Town B, then to Town C, and returns to Town A. What distance does Jill travel.

- 56.1 km
- 74.8 km
- 93.5 km
- 112.2 km



14. The picture below shows a clock. Which segment represents the intersection of planes ABC and BFE?

- $\overline{AB}$
- $\overline{BC}$
- $\overline{BF}$
- $\overline{FG}$



15. Point M is the midpoint of  $\overline{PQ}$ .  $PM = (x + 5)$  inches and  $MQ = (2x - 4)$  inches. What is the length of  $\overline{PQ}$ ?

- 9 inches
- 14 inches
- 18 inches
- 28 inches

**Section 4**—angles

Angle = \_\_\_\_\_

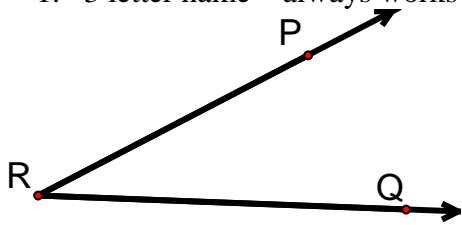
Every angle has 2 sides and 1 vertex.

Sides of the angle = \_\_\_\_\_

Vertex of the angle = \_\_\_\_\_

**Naming Angles**—there are 3 different methods used to name angles. You need to be comfortable with **all 3** different methods. (Sorry, I did not make this up.)

- 3 letter name—always works but is very long since you have to use 3 letters

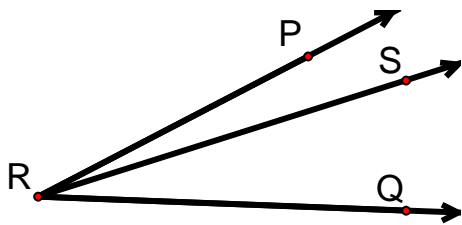


This can be called  $\angle PRQ$  or  $\angle QRP$ .

The order of the letters does not matter **as long as the vertex is the middle of the 3 letters.**

The name is read as ‘angle PRQ’ or ‘angle QRP’

- One letter name—does not work for every situation

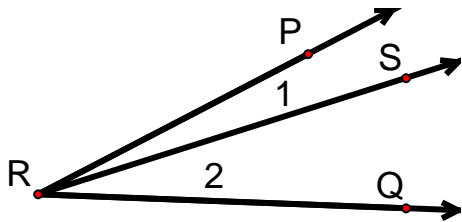


In the picture above, you can refer to  $\angle R$  since there is only one angle in that picture that has R as a vertex.

In this picture, there is no such thing as  $\angle R$  since you would not know which of the 3 different angles I was referring to.

When you use the single letter name, the angle symbol is followed by the VERTEX letter name.

- number name—also does not work in every situation



$\angle PRS$  can be referred to simply as  $\angle$  \_\_\_\_.

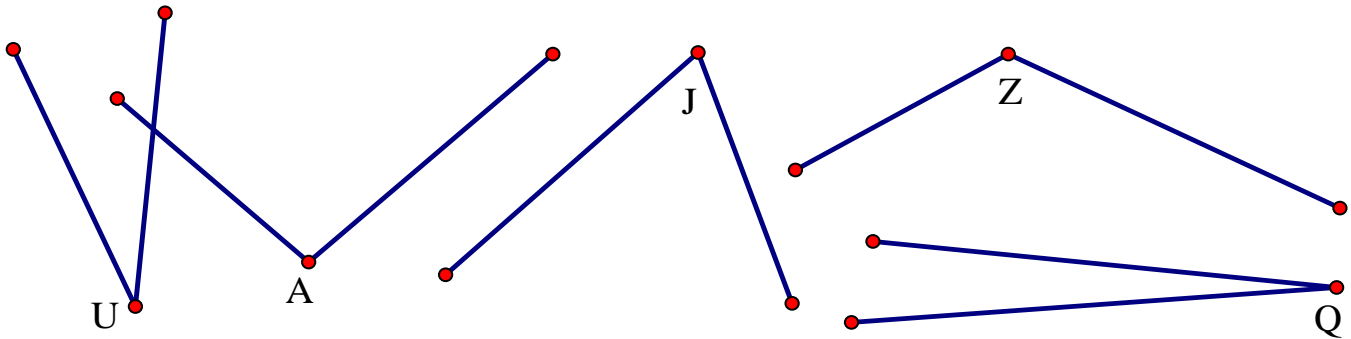
$\angle SRQ$  can be referred to simply as  $\angle$  \_\_\_\_.

$\angle PRQ$  cannot be referred to by the number method.

<p>Give 3 different names for this angle.</p>	<p>Name 3 different angles in this picture.</p>	<p>Name all the angles shown here.</p>	<p>Give 4 different names for the angle shown here.</p>
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- Measuring an angle—using a protractor should be as simple as using a ruler.
- PRACTICE—and remember that each angle should have only one measurement. You DO NOT get one measurement for each ray of the angle.

Measure these angles.



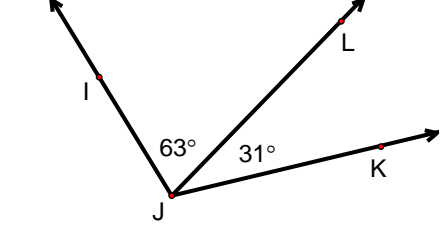
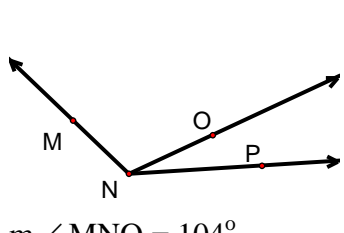
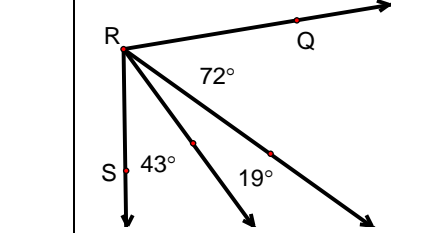
### Classifying angles

- Acute angle = any angle that measures between  $\underline{\hspace{1cm}}$ ° and  $\underline{\hspace{1cm}}$ °
- Right angle = any angle that measures exactly  $\underline{\hspace{1cm}}$ °
- Obtuse angle = any angle that measures between  $\underline{\hspace{1cm}}$ ° and  $\underline{\hspace{1cm}}$ °
- Straight angle = any angle that measures exactly  $\underline{\hspace{1cm}}$ °

$m \angle DEG = \underline{\hspace{1cm}}^\circ$	Type of angle	
$m \angle DEF = \underline{\hspace{1cm}}^\circ$	Type of angle	
$m \angle FEG = \underline{\hspace{1cm}}^\circ$	Type of angle	
$m \angle HEF = \underline{\hspace{1cm}}^\circ$	Type of angle	
$m \angle DEH = \underline{\hspace{1cm}}^\circ$	Type of angle	

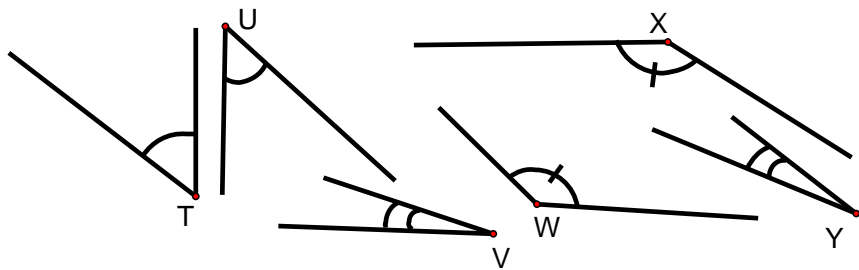
Angle measure	Picture of angle (use protractor)	Classify angle
26°		
180°		
163°		
90°		

ANGLE ADDITION POSTULATE—As long as the angles are adjacent (share a side without overlapping), you can add 2 smaller angles to get the measure of the larger angle.

 <p><math>m \angle IJK = \underline{\hspace{2cm}}^\circ</math></p>	 <p><math>m \angle MNO = 104^\circ</math>  <math>m \angle MNP = 156^\circ</math>  <math>m \angle ONP = \underline{\hspace{2cm}}^\circ</math></p>	 <p><math>m \angle SRQ = \underline{\hspace{2cm}}^\circ</math></p>
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- Congruent angles = \_\_\_\_\_

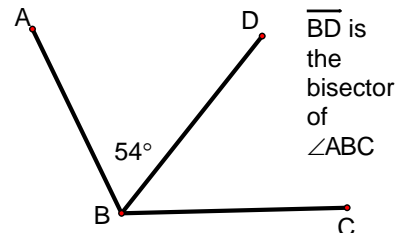
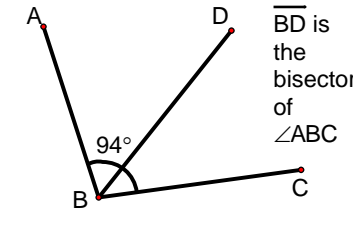
You will see pictures marked to show congruent angles.

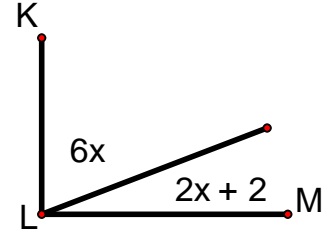
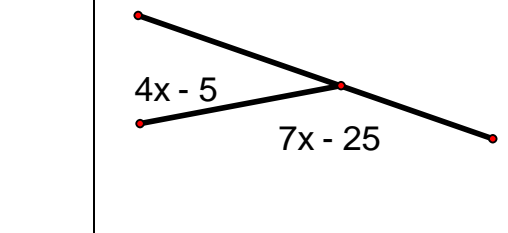
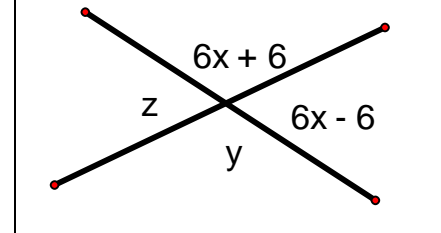


$\angle T$  and  $\angle U$  are congruent.  
 $\angle X$  and  $\angle W$  are congruent.  
 $\angle V$  and  $\angle Y$  are congruent.  
 $\angle T$  and  $\angle X$  are NOT congruent since they are marked differently.

- Angle bisector = \_\_\_\_\_

Watch carefully how the pictures get marked. There is a correct way to read them.

 <p><math>\overline{BD}</math> is the bisector of <math>\angle ABC</math></p> <p><math>m \angle ABC = \underline{\hspace{2cm}}^\circ</math>  <math>m \angle DBC = \underline{\hspace{2cm}}^\circ</math></p>	 <p><math>\overline{BD}</math> is the bisector of <math>\angle ABC</math></p> <p><math>m \angle ABD = \underline{\hspace{2cm}}^\circ</math></p>
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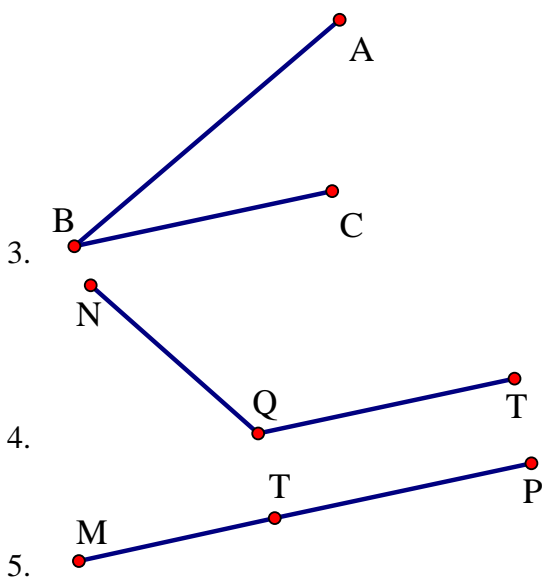
 <p><math>m \angle KLM = 90^\circ</math> <math>x = \underline{\hspace{2cm}}</math></p>	 <p><math>x = \underline{\hspace{2cm}}</math></p>	 <p><math>x = \underline{\hspace{2cm}}</math> <math>y = \underline{\hspace{2cm}}</math> <math>z = \underline{\hspace{2cm}}</math></p>
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## SKILL PRACTICE

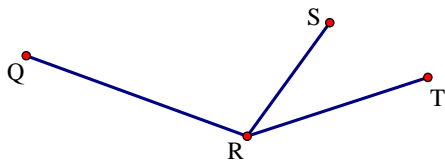
1. VOCABULARY—Sketch an example of each of the following types of angles: acute, obtuse, right, and straight.

(Problem 2 involves a picture.)

NAMING ANGLES AND ANGLE PARTS In exercises 3-5, write **3 names for the angle** shown. Then **name the vertex** and **name the sides of the angle**.



6. NAMING ANGLES—Name 3 different angles in the diagram shown.

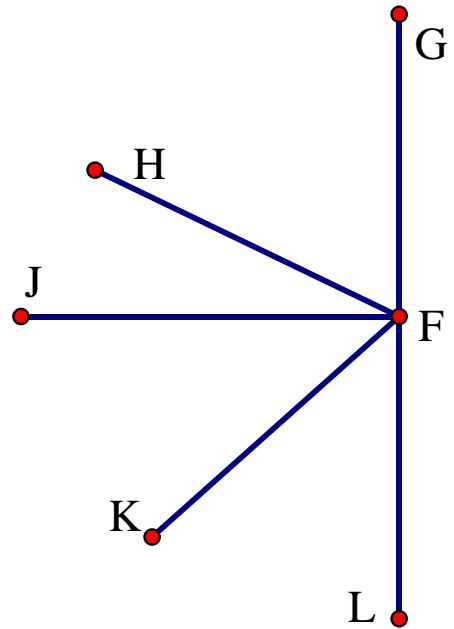


CLASSIFYING ANGLES Classify the angle with the given measure as *acute*, *right*, *obtuse*, or *straight*.

7.  $m\angle W = 180^\circ$
8.  $m\angle X = 30^\circ$
9.  $m\angle Y = 90^\circ$
10.  $m\angle Z = 95^\circ$

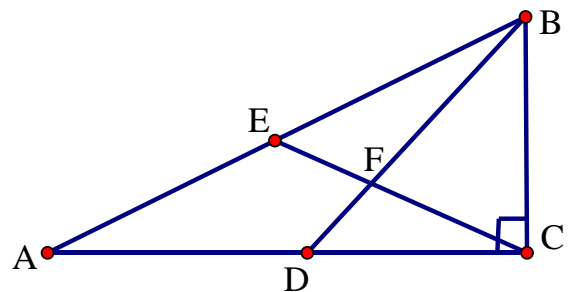
**MEASURING ANGLES** Use a **protractor** to find the measure of the given angle. Then classify the angle as *acute*, *right*, *obtuse*, or *straight*.

11.  $\angle JFL$
12.  $\angle GFH$
13.  $\angle GFK$
14.  $\angle GFL$

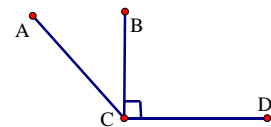


**NAMING AND CLASSIFYING** Give another name for the angle in the diagram. Tell whether the angle **appears** to be *acute*, *right*, *obtuse*, or *straight*.

15.  $\angle ACB$
16.  $\angle ABC$
17.  $\angle BFD$
18.  $\angle AEC$
19.  $\angle BDC$
20.  $\angle BEC$

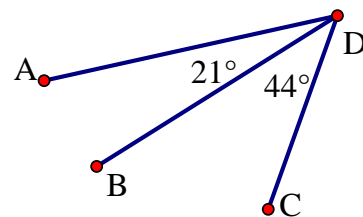
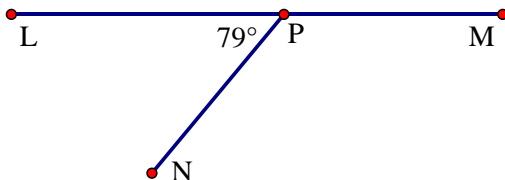
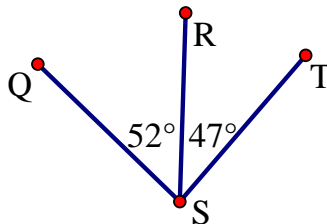


21. Which is a correct name for the obtuse angle in the diagram?
- a.  $\angle ACB$
  - b.  $\angle ACD$
  - c.  $\angle BCD$
  - d.  $\angle C$



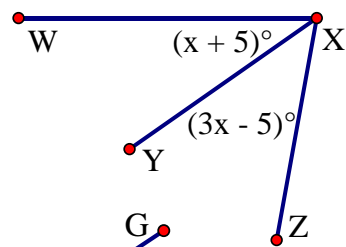
**ANGLE ADDITION POSTULATE** Find the indicated angle measure. **Do not use a protractor.**

22.  $m\angle QST = \underline{\hspace{2cm}}$
23.  $m\angle ADC = \underline{\hspace{2cm}}$
24.  $m\angle NPM = \underline{\hspace{2cm}}$

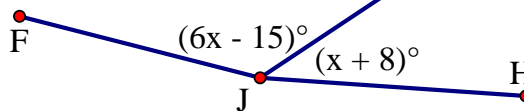


ALGEBRA Use the given information to find the indicated angle measure.

25. Given  $m\angle WXZ = 80^\circ$ , find  $m\angle YXZ$

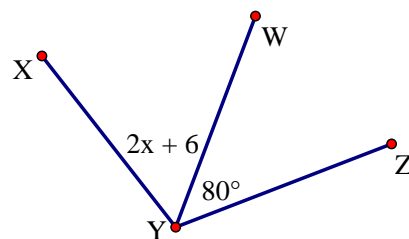


26. Given  $m\angle FJH = 168^\circ$ , find  $m\angle FJG$

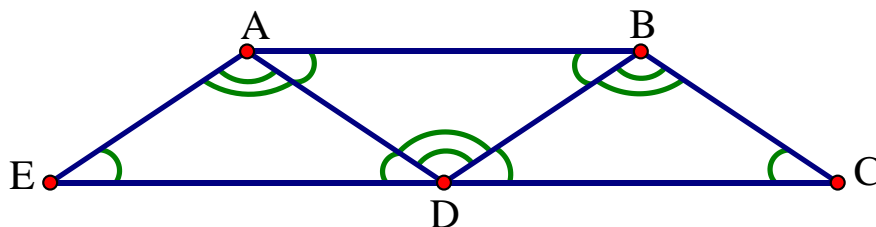


27. In the diagram, the measure of  $\angle XYZ$  is  $140^\circ$ . What is the value of  $x$ ?

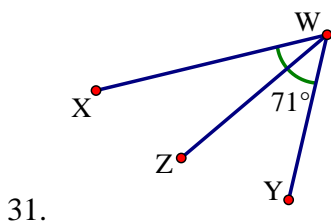
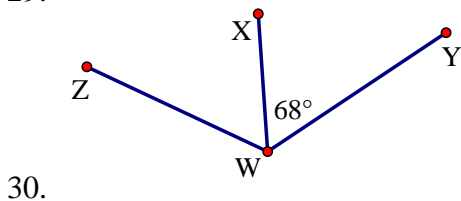
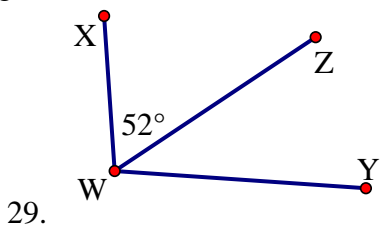
- a. 27
- b. 33
- c. 67
- d. 73



28. CONGRUENT ANGLES—In the picture below,  $m\angle AED = 34^\circ$  and  $m\angle EAD = 112^\circ$ . Identify the congruent angles in the diagram. Then find  $m\angle BDC$  and  $m\angle ADB$ .



ANGLE BISECTORS Given that  $\overline{WZ}$  bisects  $\angle XWY$ , find the 2 angle measures not given in the diagram.



32. ERROR ANALYSIS--  $\overrightarrow{KM}$  bisects  $\angle JKL$  and  $m\angle JKM = 30^\circ$ . Describe and correct the error made in stating that  $m\angle JKL = 15^\circ$ . Draw a sketch to support your answer.

FINDING ANGLE MEASURES Find the indicated angle measure.

33.  $a^\circ$

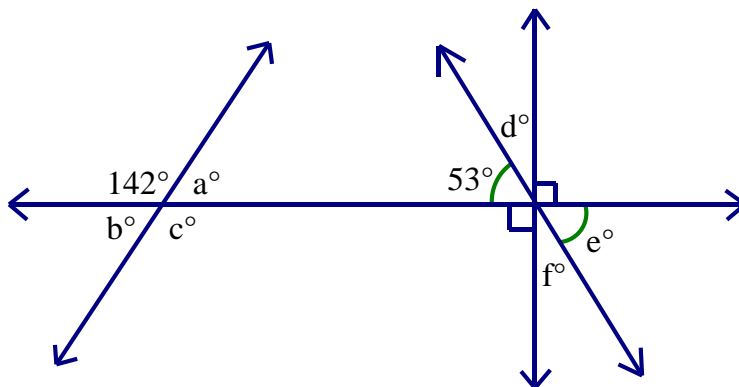
34.  $b^\circ$

35.  $c^\circ$

36.  $d^\circ$

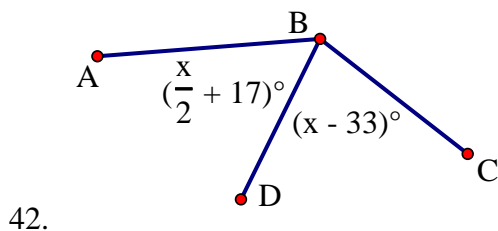
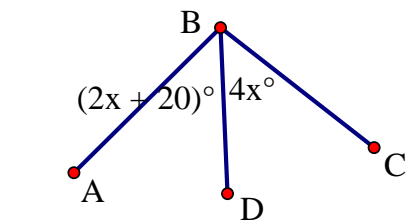
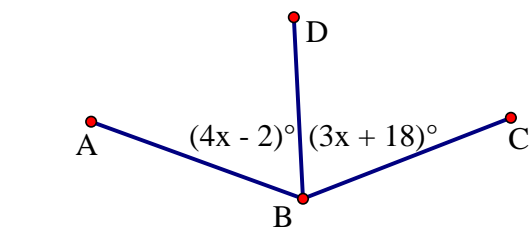
37.  $e^\circ$

38.  $f^\circ$



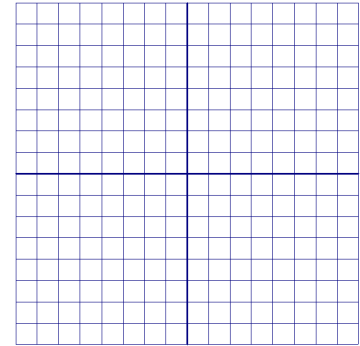
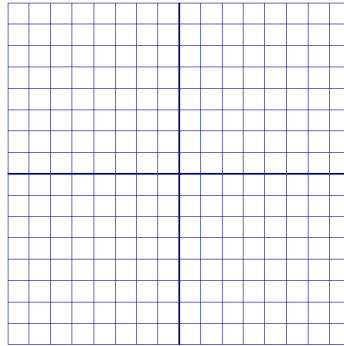
39. ERROR ANALYSIS—A student states that  $\overrightarrow{AD}$  can bisect  $\angle AGC$ . Describe and correct the student's error. Draw a sketch to support your answer.

ALGEBRA In each diagram,  $\overrightarrow{BD}$  bisects  $\angle ABC$ . Find  $m\angle ABC$ .



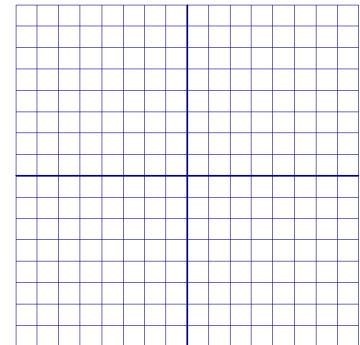
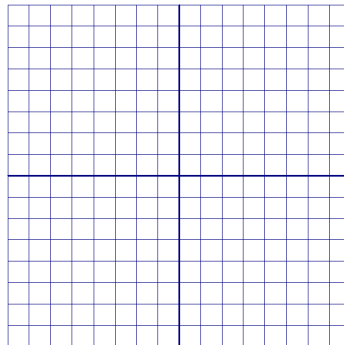
43. SHORT RESPONSE—You are measuring  $\angle PQR$  with a protractor. When you line up  $\overline{QR}$  with the  $20^\circ$  mark,  $\overline{QP}$  lines up with the  $80^\circ$  mark. Then you move the protractor so that  $\overline{QR}$  lines up with the  $15^\circ$  mark. What mark does  $\overline{QP}$  line up with? *Explain.*

ALGEBRA Plot the points in a coordinate plane and draw  $\angle ABC$ . Classify the angle. Then give the coordinates of a point that lies in the interior of the angle.



44. A (3, 3), B (0, 0), C (3, 0)

45. A (-5, 4), B (1, 4), C (-2, -2)



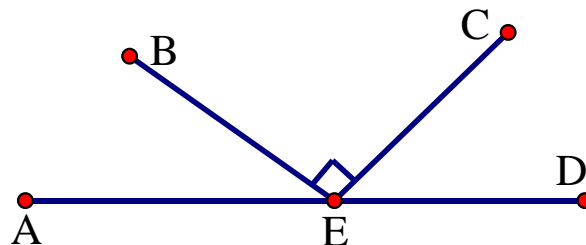
46. A (-5, 2), B (-2, -2), C (4, -3)

47. A (-3, -1), B (2, 1), C (6, -2)

48. ALGEBRA—Let  $(2x - 12)^\circ$  represent the measure of an acute angle. What are the possible values of  $x$ ?

49. CHALLENGE— $\overline{SQ}$  bisects  $\angle RST$ ,  $\overline{SP}$  bisects  $\angle RSQ$ , and  $\overline{SV}$  bisects  $\angle RSP$ . The measure of  $\angle VSP$  is  $17^\circ$ . Find  $m\angle TSQ$ . *Explain.*

50. FINDING MEASURES—In the diagram,  $m\angle AEB = \frac{1}{2} m\angle CED$ , and  $\angle AED$  is a straight angle. Find  $m\angle AEB$  and  $m\angle CED$ .



**PROBLEM SOLVING**

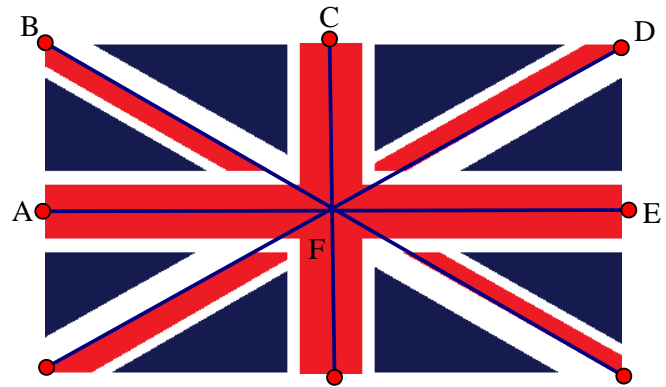
(problems 51-61 involve pictures)

61. EXTENDED RESPONSE—In the flag shown,  $\angle AFE$  is a straight angle and  $\overline{FC}$  bisects  $\angle AFE$  and  $\angle BFD$ .

a. Which angles are acute?

Which angles are obtuse?

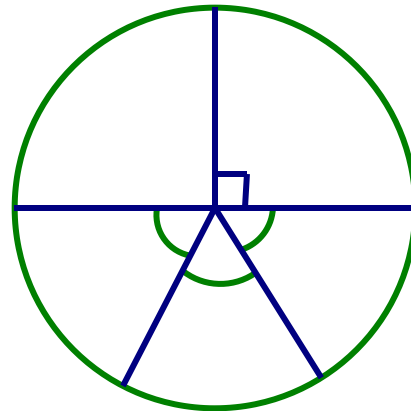
Which angles are right?



b. Identify the congruent angles.

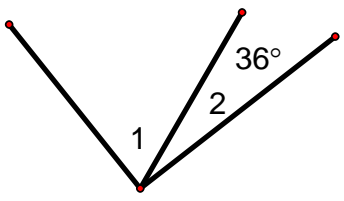
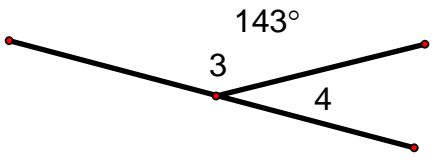
c. If  $m\angle AFB = 26^\circ$ , find  $m\angle DFE$ ,  $m\angle BFC$ ,  $m\angle CFD$ ,  $m\angle AFC$ ,  $m\angle AFD$ , and  $m\angle BFD$ .  
*Explain.*

62. CHALLENGE—Create a set of data that could be represented by the circle graph shown. *Explain* your reasoning.



**Section 5**—angle relationships

- Complementary angles = \_\_\_\_\_
- Supplementary angles = \_\_\_\_\_

<p><math>\angle 1</math> and <math>\angle 2</math> are complementary.</p> 	<p><math>\angle 3</math> and <math>\angle 4</math> are supplementary.</p> 
<p><math>m\angle 1 = \underline{\hspace{2cm}}^\circ</math></p>	<p><math>m\angle 4 = \underline{\hspace{2cm}}^\circ</math></p>

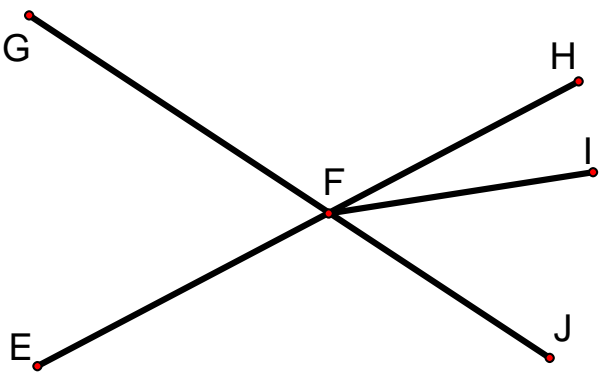
Some angles do not have complements.  
Some angles do not have supplements.

Angle	Complement	Supplement
$36^\circ$		
$137^\circ$		
$26.3^\circ$		
$90^\circ$		
$180^\circ$		

Any angle that is greater than or equal to \_\_\_\_\_ degrees does not have a complementary angle.

Any angle that is \_\_\_\_\_ degrees does not have a supplementary angle.

- Adjacent angles = \_\_\_\_\_
- Linear pairs of angles = \_\_\_\_\_
- Vertical angles = \_\_\_\_\_

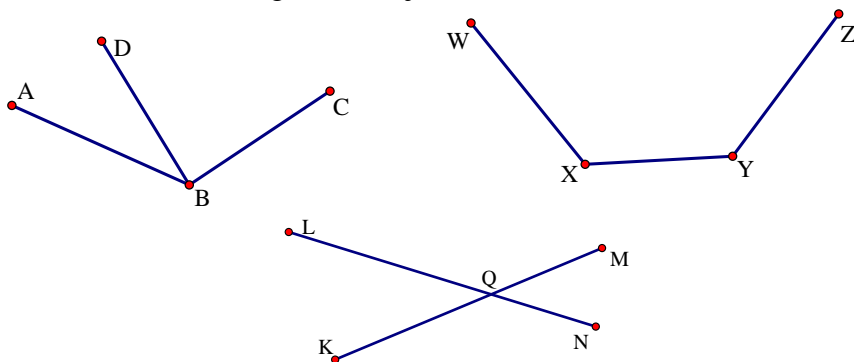
<p>List 2 pair of vertical angles</p> <p><math>\angle</math> ___ and <math>\angle</math> ___</p> <p><math>\angle</math> ___ and <math>\angle</math> ___</p>	
<p>List 6 pair of adjacent angles</p> <p><math>\angle</math> ___ and <math>\angle</math> ___      <math>\angle</math> ___ and <math>\angle</math> ___</p> <p><math>\angle</math> ___ and <math>\angle</math> ___      <math>\angle</math> ___ and <math>\angle</math> ___</p> <p><math>\angle</math> ___ and <math>\angle</math> ___      <math>\angle</math> ___ and <math>\angle</math> ___</p>	
<p>List 3 linear pairs of angles</p> <p><math>\angle</math> ___ and <math>\angle</math> ___</p> <p><math>\angle</math> ___ and <math>\angle</math> ___</p> <p><math>\angle</math> ___ and <math>\angle</math> ___</p>	

**SKILL PRACTICE**

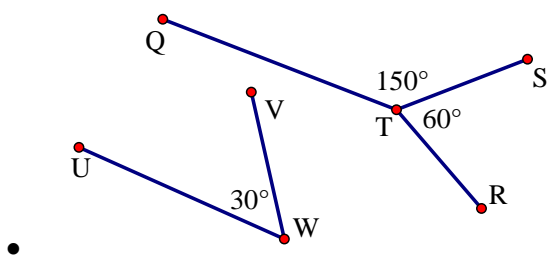
- VOCABULARY—Sketch an example of adjacent angles that are complementary. Are all complementary angles adjacent angle? Explain.
- WRITING—Are all linear pairs supplementary angles? Are all supplementary angles linear pairs? Explain.

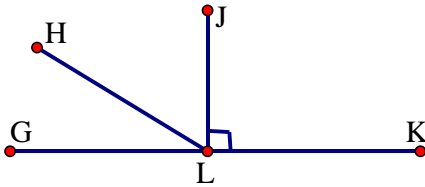
IDENTIFYING ANGLES Tell whether the indicated angles are adjacent.

- $\angle ABD$  and  $\angle DBC$
- $\angle WXY$  and  $\angle XYZ$
- $\angle LQM$  and  $\angle NQM$



IDENTIFYING ANGLES Name a pair of complementary angles and a pair of supplementary angles.





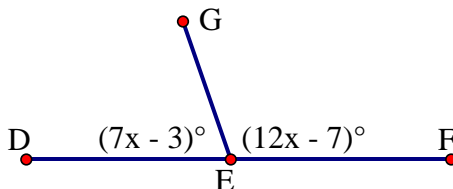
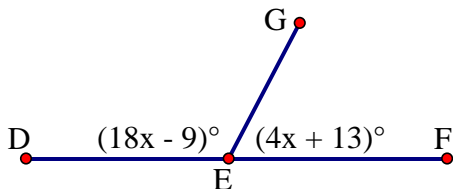
COMPLEMENTARY ANGLES  $\angle 1$  and  $\angle 2$  are complementary angles. Given the measure of  $\angle 1$ , find  $m\angle 2$ .

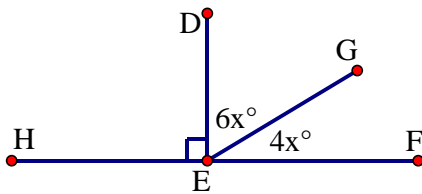
- $m\angle 1 = 60^\circ$
- $m\angle 1 = 21^\circ$
- $m\angle 1 = 89^\circ$
- $m\angle 1 = 5^\circ$

SUPPLEMENTARY ANGLES  $\angle 1$  and  $\angle 2$  are supplementary angles. Given the measure of  $\angle 1$ , find  $m\angle 2$ .

- $m\angle 60^\circ$
- $m\angle 155^\circ$
- $m\angle 130^\circ$
- $m\angle 27^\circ$
- The arm of a crossing gate moves  $37^\circ$  from vertical. How many more degrees does the arm have to move so that it is horizontal?
  - a.  $37^\circ$
  - b.  $53^\circ$
  - c.  $90^\circ$
  - d.  $143^\circ$

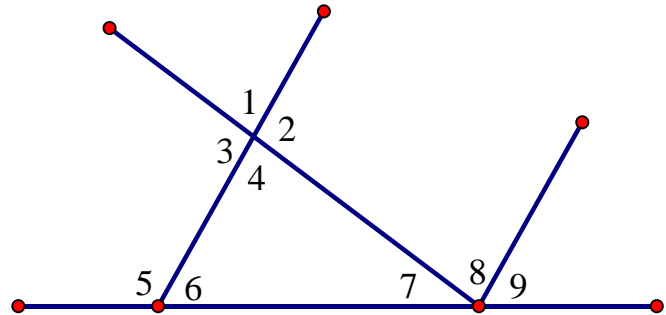
ALGEBRA Find  $m\angle DEG$  and  $m\angle GEF$ .



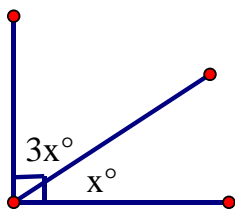


IDENTIFYING ANGLE PAIRS Use the diagram below. Tell whether the angles are *vertical angles*, a *linear pair* of angles, or *neither*.

- $\angle 1$  and  $\angle 4$
- $\angle 1$  and  $\angle 2$
- $\angle 3$  and  $\angle 5$
- $\angle 2$  and  $\angle 3$
- $\angle 7$ ,  $\angle 8$ , and  $\angle 9$
- $\angle 5$  and  $\angle 6$
- $\angle 6$  and  $\angle 7$
- $\angle 5$  and  $\angle 9$



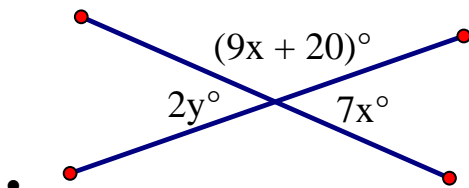
- ALGEBRA—2 angles form a linear pair. The measure of one angle is 4 times the measure of the other angle. Find the measure of each angle.
- ERROR ANALYSIS—*Describe* and correct the error made in finding the value of  $x$ .

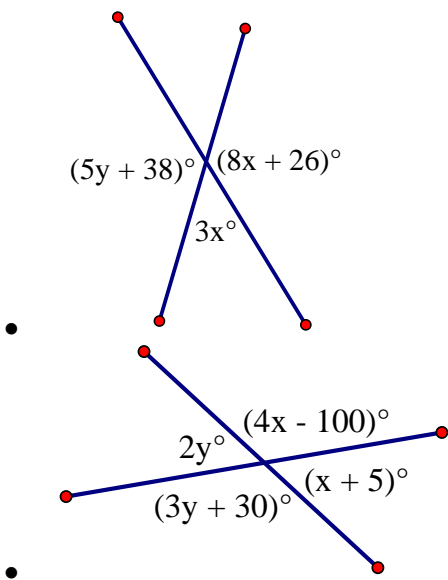


$$\begin{aligned} x^\circ + 3x^\circ &= 180^\circ \\ 4x^\circ &= 180^\circ \\ x &= 45^\circ \end{aligned}$$

- The measure of one angle is  $24^\circ$  greater than the measure of its complement. What are the measures of the angles?

ALGEBRA Find the values of  $x$  and  $y$ .





**REASONING** Tell whether the statement is *always*, *sometimes*, or *never* true. *Explain* your reasoning.

- An obtuse angle has a complement.
- A straight angle has a complement.
- An angle has a supplement.
- The complement of an acute angle is an acute angle.
- The supplement of an acute angle is an obtuse angle.

**FINDING ANGLES**  $\angle A$  and  $\angle B$  are complementary. Find  $m\angle A$  and  $m\angle B$ .

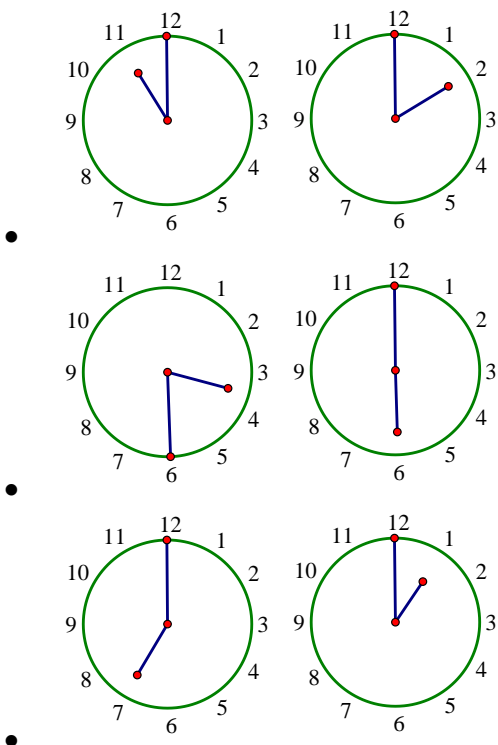
- $m\angle A = (3x + 2)^\circ$ ,  $m\angle B = (x - 4)^\circ$
- $m\angle A = (15x + 3)^\circ$ ,  $m\angle B = (5x - 13)^\circ$
- $m\angle A = (11x + 24)^\circ$ ,  $m\angle B = (x + 18)^\circ$

**FINDING ANGLES**  $\angle A$  and  $\angle B$  are supplementary. Find  $m\angle A$  and  $m\angle B$ .

- $m\angle A = (8x + 100)^\circ$ ,  $m\angle B = (2x + 50)^\circ$
- $m\angle A = (2x - 20)^\circ$ ,  $m\angle B = (3x + 5)^\circ$
- $m\angle A = (6x + 72)^\circ$ ,  $m\angle B = (2x + 28)^\circ$
- **CHALLENGE**—You are given that  $\angle GHJ$  is a complement of  $\angle RST$  and  $\angle RST$  is a supplement of  $\angle ABC$ . Let  $m\angle GHJ$  be  $x^\circ$ . What is the measure of  $\angle ABC$ ? *Explain* your reasoning.

## PROBLEM SOLVING

IDENTIFYING ANGLES Tell whether the 2 angles shown are *complementary*, *supplementary*, or *neither*.



(Problems 49-54 involve photographs)

55. MULTIPLE REPRESENTATIONS—Let  $x^\circ$  be an angle measure. Let  $y_1^\circ$  be the measure of a complement of the angle and let  $y_2^\circ$  be the measure of a supplement of the angle.

a. **Writing an Equation**—Write equations for  $y_1$  as a function of  $x$ , and for  $y_2$  as a function of  $x$ . What is the domain for each function? *Explain*.

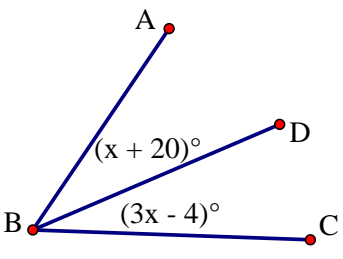
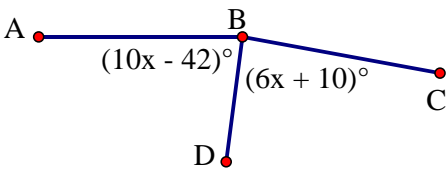
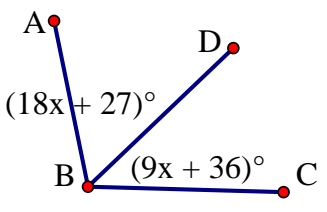
b. **Drawing a Graph**—Graph each function and *describe* each range.

56. CHALLENGE—The sum of the measures of 2 complementary angles exceed the difference of their measures by  $86^\circ$ . Find the measure of each angle. *Explain* how you found the angle measures.

QUIZ

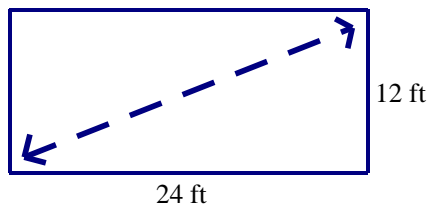
name \_\_\_\_\_

In each diagram,  $\overrightarrow{BD}$  bisects  $\angle ABC$ . Find  $m\angle ABD$  and  $m\angle DBC$ .

1. 
2. 
3. 

Find the measure of (a) the complement and (b) the supplement of  $\angle 1$ .

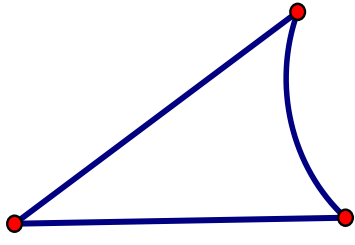
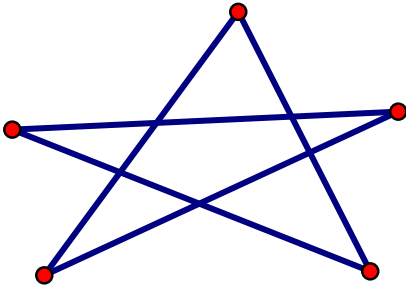
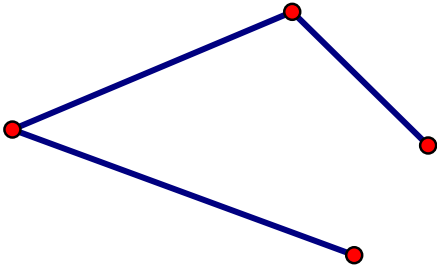
4.  $m\angle 1 = 47^\circ$
5.  $m\angle 1 = 19^\circ$
6.  $m\angle 1 = 75^\circ$
7.  $m\angle 1 = 2^\circ$
8. Anna swims diagonal laps in the pool shown. About how many laps must she complete to swim 0.5 mile. (One mile is 5280 feet.)
  - a. 73
  - b. 98
  - c. 127
  - d. 197



**Section 6**—polygons

All polygons are 2 dimensional shapes with 3 common characteristics:

1. \_\_\_\_\_
2. \_\_\_\_\_
3. \_\_\_\_\_

 <p style="text-align: center;">This is NOT a polygon because</p>	 <p style="text-align: center;">This is NOT a polygon because</p>	 <p style="text-align: center;">This is NOT a polygon because</p>
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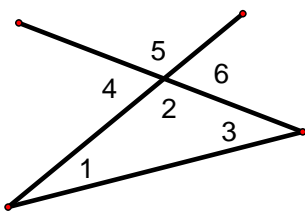
**Parts of a polygon**

Side of a polygon = \_\_\_\_\_

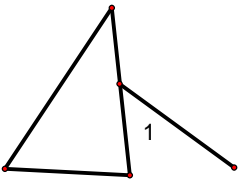
Vertex of a polygon = \_\_\_\_\_

- Interior angles = \_\_\_\_\_
- Exterior angles = \_\_\_\_\_

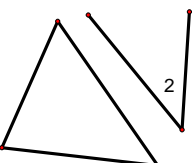
All angles outside a polygon are NOT exterior angles.



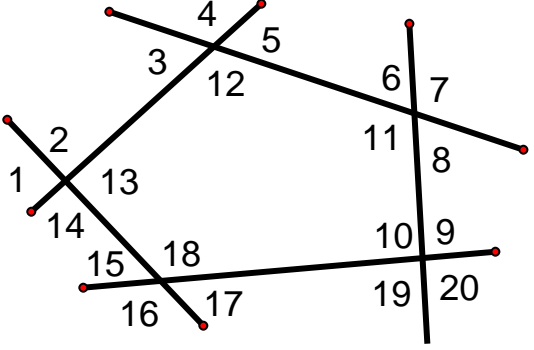
$\angle 1$ ,  $\angle 2$ , and  $\angle 3$  are all interior angles  
 $\angle 4$  and  $\angle 6$  are exterior angles  
 $\angle 5$  is neither an interior nor an exterior angle because \_\_\_\_\_



$\angle 1$  is NOT an exterior angle because \_\_\_\_\_.



$\angle 2$  is NOT an exterior angle because \_\_\_\_\_.

	<p>List ALL the interior angles</p>    <p>List ALL the exterior angles.</p>
---	---

- convex polygon = \_\_\_\_\_
- concave polygon = \_\_\_\_\_

Number of sides	Name	Picture	
		Convex example	Concave example
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
n			

Regular = \_\_\_\_\_

- Equilateral = \_\_\_\_\_
- Equiangular = \_\_\_\_\_

With triangles, equilateral and equiangular go together (you cannot have one without the other).

An equilateral triangle is always equiangular.

An equiangular triangle is always equilateral.

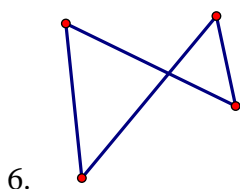
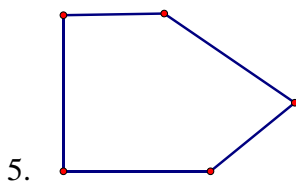
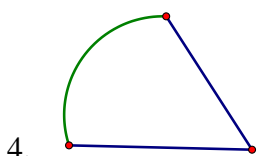
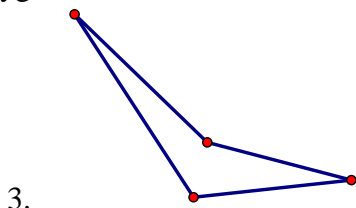
With other shapes, this is not always the case.

Draw a convex quadrilateral that is equilateral but not equiangular.	Draw a convex quadrilateral that is equiangular but not equilateral.	Draw a convex quadrilateral that is regular.
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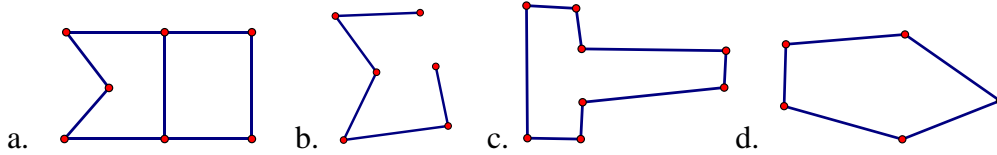
### SKILL PRACTICE

1. VOCABULARY—*Explain* what is meant by the term  $n$ -gon.
2. WRITING—Imagine that you can tie a string tightly around a polygon. If the polygon is convex, will the length of the string be equal to the distance around the polygon? What if the polygon is concave? *Explain*.

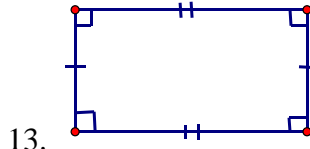
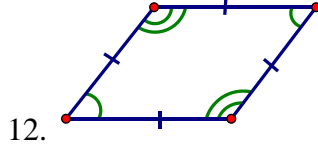
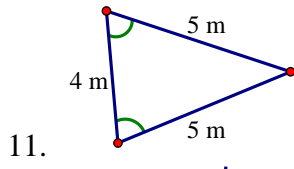
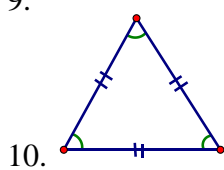
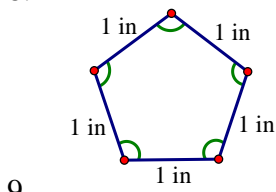
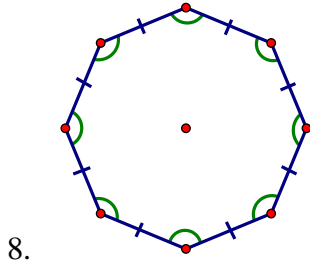
**IDENTIFYING POLYGONS** Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is *convex* or *concave*.



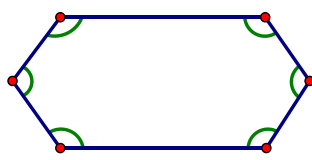
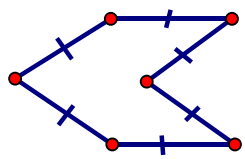
7. Which of these figures is a concave polygon



**CLASSIFYING** Classify the polygon by the number of sides. Tell whether the polygon is *equilateral*, *equiangular*, or *regular*. Explain your reasoning.



14. **ERROR ANALYSIS**—Two students were asked to draw a regular hexagon, as shown below. Describe the error made by each student.



15. **ALGEBRA**—The lengths (in inches) of 2 sides of a regular pentagon are represented by the expressions  $5x - 27$  and  $2x - 6$ . Find the length of a side of the pentagon.

16. ALGEBRA—The expressions  $(9x + 5)^\circ$  and  $(11x - 25)^\circ$  represent the measures of 2 angles of a regular nonagon. Find the measure of an angle of the nonagon.

17. ALGEBRA—The expressions  $3x - 9$  and  $23 - 5x$  represent the lengths (in feet) of 2 sides of an equilateral triangle. Find the length of a side.

USING PROPERTIES Tell whether the statement is *always*, *sometimes*, or *never* true.

18. A triangle is convex.

19. A decagon is regular.

20. A regular polygon is equiangular.

21. A circle is a polygon.

22. A polygon is a plane figure.

23. A concave polygon is regular.

DRAWING Draw a figure that fits the description.

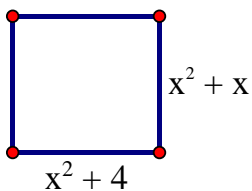
24. A triangle that is not regular

25. A concave quadrilateral

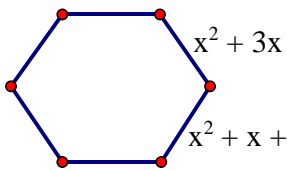
26. A pentagon that is equilateral but not equiangular

27. An octagon that is equiangular but not equilateral

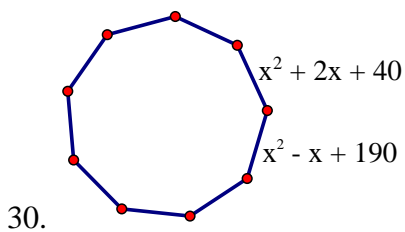
ALGEBRA Each figure is a regular polygon. Expressions are given for 2 side lengths. Find the value of  $x$ .



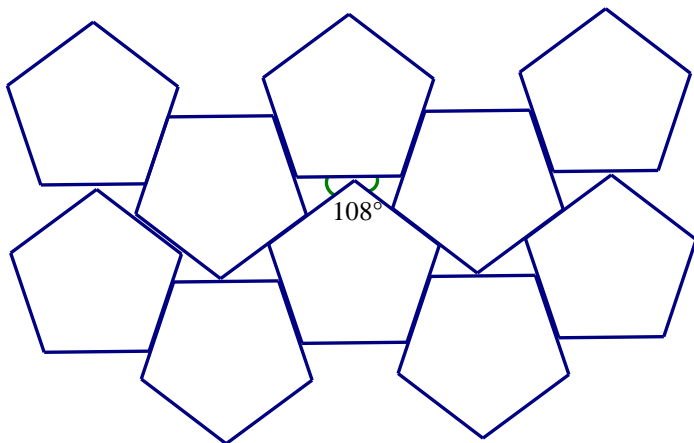
28.



29.



31. CHALLENGE—Regular pentagonal tiles and triangular tiles are arranged in the pattern shown. The pentagonal tiles are all the same size and shape and the triangular tiles are all the same size and shape. Find the angle measures of the triangular tiles. *Explain* your reasoning.

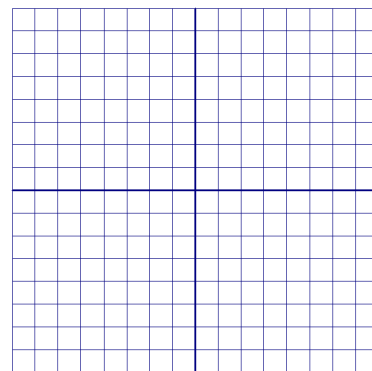


**PROBLEM SOLVING**

(Problems 32-36 involve photographs.)

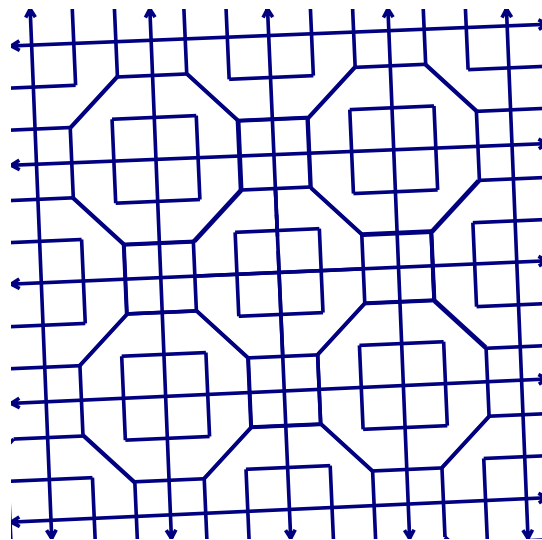
37. Two vertices of a regular quadrilateral are A (0, 4) and B (0, -4). Which of the following could be the other 2 vertices?

- a. C (4, 4) and D (4, -4)
- b. C (-4, 4) and D (-4, -4)
- c. C (8, -4) and D (8, 4)
- d. C (0, 8) and D (0, -8)



38. MULTI-STEP PROBLEM—The diagram shows the design of a lattice made in China in 1850.

- a. Sketch 5 different polygons you see in the diagram. Classify each polygon by the number of sides.
- b. Tell whether each polygon you sketched is concave or convex, and whether the polygon appears to be equilateral, equiangular, or regular.



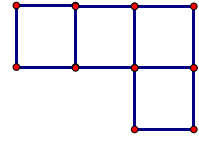
(Problem 39 involves a photograph)

40. EXTENDED RESPONSE—A segment that joins 2 nonconsecutive vertices of a polygon is called *diagonal*. For example, a quadrilateral has 2 diagonals as shown below.

Type of Polygon	Diagram	Number of Sides	Number of Diagonals
Quadrilateral		4	2
Pentagon			
Hexagon			
Heptagon			

- a. Complete the table. *Describe* any patterns you see.
  
  - b. How many diagonals does an octagon have? a nonagon? *Explain*.
  
  - c. The expression  $\frac{n(n-3)}{2}$  can be used to find the number of diagonals in an n-gon. Find the number of diagonals in a 60-gon.
41. LINE SYMMETRY—A figure has *line symmetry* if it can be folded over exactly onto itself. The fold line is called the *line of symmetry*. A regular quadrilateral had 4 lines of symmetry, as shown. Find the number of lines of symmetry in each polygon.
- a. A regular triangle
  
  - b. A regular pentagon
  
  - c. A regular hexagon
  
  - d. A regular octagon

42. CHALLENGE—The diagram shows 4 identical squares lying edge-to-edge. Sketch all the different ways you can arrange 4 squares edge-to-edge.



Sketch all the different ways you can arrange 5 identical squares edge-to-edge.

43. The radius of Cylinder A is 3 times the radius of Cylinder B. The heights of the cylinders are equal. How many times greater is the volume of Cylinder A than the volume of Cylinder B?
- a. 3
  - b. 6
  - c. 9
  - d. 27

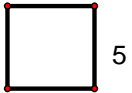
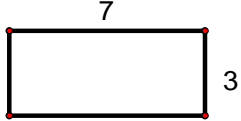
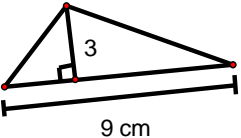
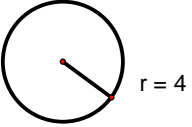
## Section 7

Area = \_\_\_\_\_

Perimeter = \_\_\_\_\_

Circumference = \_\_\_\_\_

- Formulas for AREA
  - Square = \_\_\_\_\_
  - Rectangle = \_\_\_\_\_
  - Triangle = \_\_\_\_\_
  - Circle = \_\_\_\_\_ where  $\pi$  is approximately \_\_\_\_\_
- Perimeter—add all the side measures
- Circumference = \_\_\_\_\_

<p>Find the perimeter and area of this square.</p>  <p>P = _____ A = _____</p>	<p>Find the perimeter and area of this rectangle.</p>  <p>P = _____ A = _____</p>
<p>Find the perimeter and area of this triangle.</p>  <p>P = _____ A = _____</p>	<p>Find the circumference and area of this circle.</p>  <p>C = _____ <math>\approx</math> _____ A = _____ <math>\approx</math> _____</p>

The general rule in math is that if the person who wrote the problem went to the trouble of putting **units on the problem**, you are expected to **put units on the answer**.

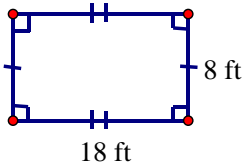
- Units on perimeter/circumference = \_\_\_\_\_
- Units on area = \_\_\_\_\_

### SKILL PRACTICE

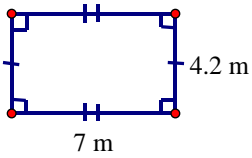
- VOCABULARY—How are the diameter and radius of a circle related?
- WRITING—*Describe* a real-world situation in which you would need to find a perimeter, and a situation in which you would need to find an area. What measurement units would you use in each situation?

- ERROR ANALYSIS—Describe and correct the error made in finding the area of a triangle with height 9 ft and a base of 52 ft.  $A = 52(9) = 468 \text{ ft}^2$

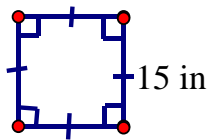
PERIMETER AND AREA Find the perimeter and the area of the shaded figure.



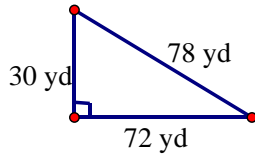
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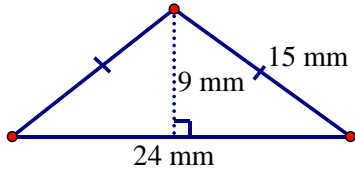
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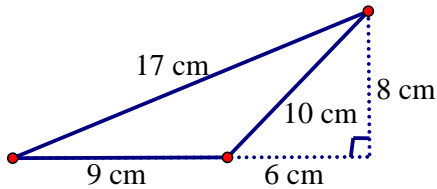
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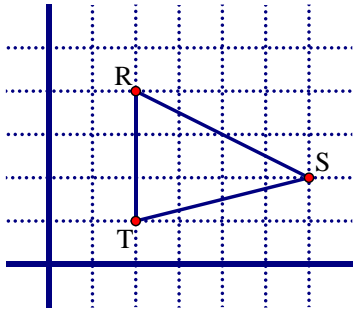
- DRAWING A DIAGRAM—The base of a triangle is 32 ft. Its height is  $16 \frac{1}{2}$  ft. Sketch the triangle and find its area.

CIRCUMFERENCE AND AREA Use the given diameter  $d$  or radius  $r$  to find the circumference and area of the circle. Round to the nearest tenth.

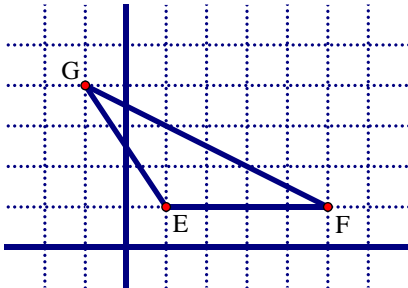
- $d = 27 \text{ cm}$
- $d = 5 \text{ in}$
- $r = 12.1 \text{ cm}$
- $r = 3.9 \text{ cm}$

- **DRAWING A DIAGRAM**—The diameter of a circle is 18.9 cm. Sketch the circle and find its circumference and area. Round your answers to the nearest tenth.

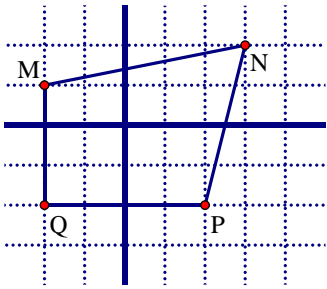
**DISTANCE FORMULA** Find the perimeter of the figure. Round to the nearest tenth of a unit.



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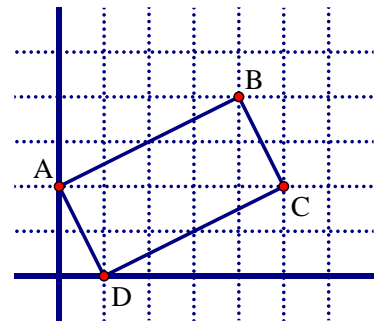


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- What is the approximate area (in square units) of the rectangle shown at the right?
  - 6.7
  - 8.0
  - 9.0
  - 10.0



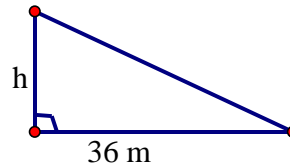
**CONVERTING UNITS** Complete each statement.

- $187 \text{ cm}^2 = \underline{\hspace{2cm}} \text{ m}^2$
- $13 \text{ ft}^2 = \underline{\hspace{2cm}} \text{ yd}^2$
- $18 \text{ in}^2 = \underline{\hspace{2cm}} \text{ ft}^2$
- $8 \text{ km}^2 = \underline{\hspace{2cm}} \text{ m}^2$
- $12 \text{ yd}^2 = \underline{\hspace{2cm}} \text{ ft}^2$

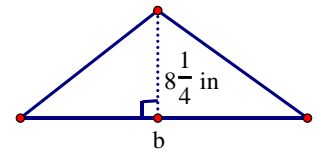
- $24 \text{ ft}^2 = \underline{\hspace{2cm}} \text{ in}^2$
- A triangle has an area of 2.25 square feet. What is the area of the triangle in square inches?
  - $27 \text{ in}^2$
  - $54 \text{ in}^2$
  - $144 \text{ in}^2$
  - $324 \text{ in}^2$

**UNKNOWN MEASURES** Use the information about the figure to find the indicated measure.

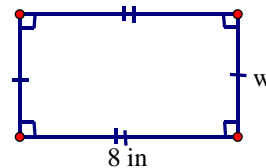
- Area =  $261 \text{ m}^2$ ; find the height  $h$ .



- Area =  $66 \text{ in}^2$ ; find the base  $b$ .



- Perimeter = 25 in; find the width  $w$ .

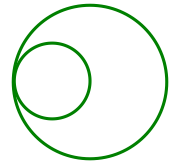


- **UNKNOWN MEASURE**—the width of a rectangle is 17 inches. Its perimeter is 102 inches. Find the length of the rectangle.
- **ALGEBRA**—The area of a rectangle is 18 square inches. The length of the rectangle is twice its width. Find the length and width of the rectangle.
- **ALGEBRA**—The area of a triangle is 27 square feet. Its height is 3 times the length of its base. Find the height and base of the triangle.
- **ALGEBRA**—Let  $x$  represent the side length of a square. Find a regular polygon with side length  $x$  whose perimeter is twice the perimeter of the square. Find a regular polygon with side length  $x$  whose perimeter is 3 times the length of the square. *Explain* your thinking.

**FINDING SIDE LENGTHS** find the side length of the square with the given area. Write your answer as a radical in simplest form.

- $A = 184 \text{ in}^2$
- $A = 346 \text{ in}^2$
- $A = 1008 \text{ m}^2$

- $A = 1050 \text{ km}^2$
- **SHORT RESPONSE**—In the diagram, the diameter of the small circle is half the diameter of the large circle. What fraction of the area of the large circle is *not* covered by the small circle? *Explain.*

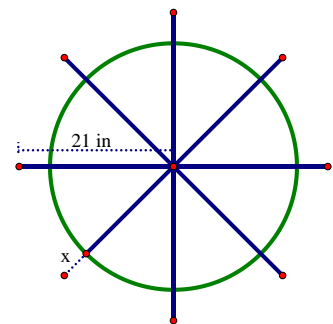


- **CHALLENGE**—The area of a rectangle is  $30 \text{ cm}^2$  and its perimeter is 26 cm. Find the length and width of the rectangle.

### PROBLEM SOLVING

- **WATER LILIES**—The giant Amazon water lily has a lily pad that is shaped like a circle. Find the circumference and area of a lily pad with a diameter of 60 inches. Round your answer to the nearest tenth.
- **LAND**—You are planting grass on a rectangular plot of land. You are also building a fence around the edge of the plot. The plot is 45 yards long and 30 yards wide. How much are you need to cover with grass seed? How many feet of fencing do you need?
- **MULTI-STEP PROBLEM**—Chris is installing a solar panel. The maximum amount of power the solar panel can generate in a day depends in part on its area. On a sunny day in the city where Chris lives, each square meter of panel can generate up to 125 watts of power. The flat rectangular panel is 84 cm long and 54 cm wide.
  - Find the area of the solar panel in square meters.
  - What is the maximum amount of power (in watts) that the panel could generate if its area was 1 square meter? 2 square meters? *Explain.*
  - Estimate the maximum amount of power Chris's solar panel can generate. *Explain* your reasoning.

- **MULTI-STEP PROBLEM**—The 8 spokes of a ship's wheel are joined at the wheel's center and pass through a large wooden circle, forming handles on the outside of the circle. From the wheel's center to the tip of the handle, each spoke is 21 inches long.
  - The circumference of the large wooden circle is 94 inches. Find the radius of the outer edge of the circle to the nearest inch.



b. Find the length  $x$  of the handle on the wheel. *Explain.*

- **MULTIPLE REPRESENTATIONS**—Let  $x$  represent the length of a side of a square. Let  $y_1$  and  $y_2$  represent the perimeter and area if that square.

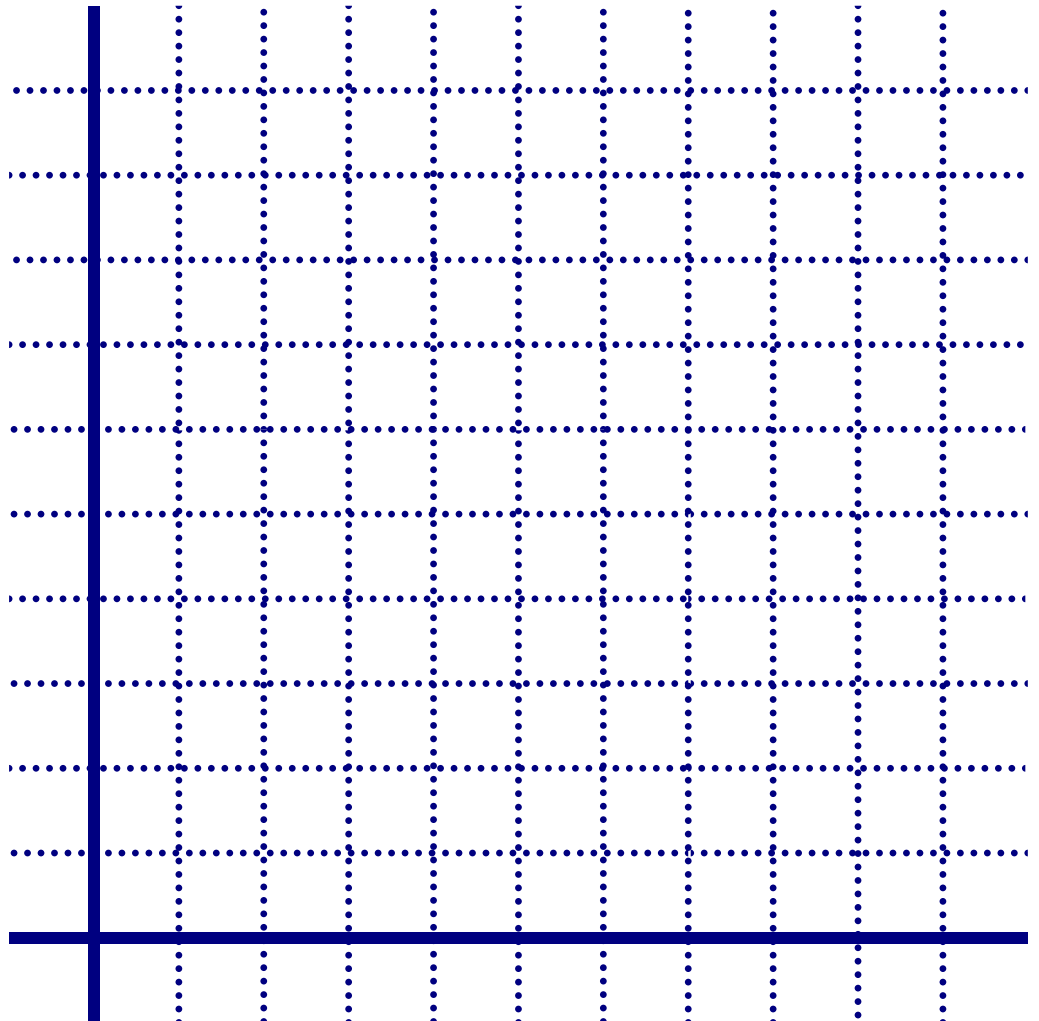
a. Complete the table.

Length ( $x$ )	1	2	5	10	25
Perimeter ( $y_1$ )					
Area ( $y_2$ )					

b. Use the completed table to write 2 sets of ordered pairs:  $(x, y_1)$  and  $(x, y_2)$ . Graph each set of ordered pairs.

$x$	$y_1$

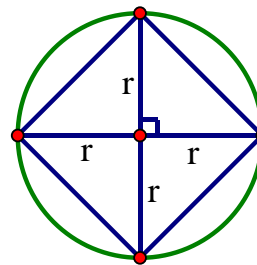
$x$	$y_2$



c. Describe any patterns you see in the table from part (a) and in the graph from part (b).

(Problems 45-46 involve photographs.)

47. CHALLENGE—In the diagram at the right, how many times as great is the area of the circle as the area of the square? *Explain* your reasoning.

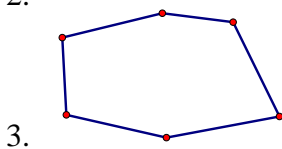
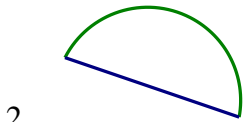
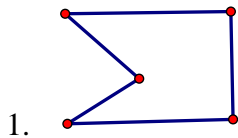


48. ALGEBRA—You have 30 yards of fencing with which to make a rectangular pen. Let  $x$  be the length of the pen.
- Write an expression for the width of the pen in terms of  $x$ . Then write a formula for the area  $y$  of the pen in terms of  $x$ .
  - You want the pen to have the greatest possible area. What length and width should you use? *Explain* your reasoning.

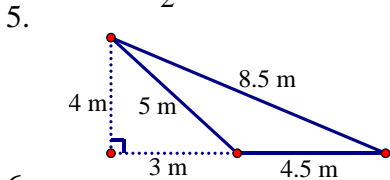
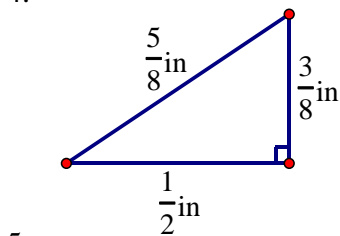
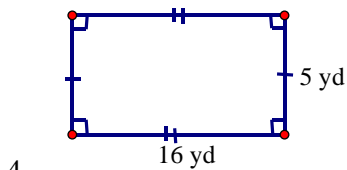
QUIZ

name \_\_\_\_\_

Tell whether the figure is a polygon. If it is not, explain why. If it is a polygon, tell whether it is convex or concave.



Find the perimeter and area of the shaded figure.



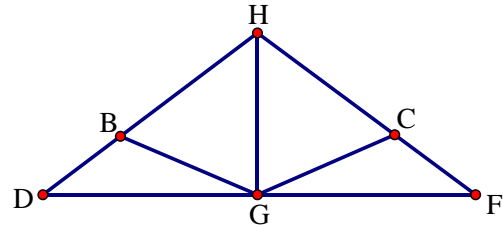
7. GARDENING—You are spreading wood chips on a rectangular garden. The garden is  $3\frac{1}{2}$  yards long and  $2\frac{1}{2}$  yards wide. One bag of wood chips covers 10 square feet. How many bags of wood chips do you need?

8. ROOFING—Jane is covering the roof of a shed with shingles. The roof is a rectangle that is 4 yards long and 3 yards wide. Asphalt shingles cost \$0.75 per square foot and wood shingles cost \$1.15 per square foot. How much more would Jane pay to use wood shingles instead of asphalt shingles?

- a. \$4.80
- b. \$43.20
- c. \$14.40
- d. 50.09

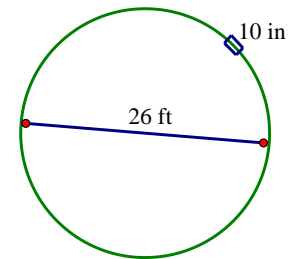
(Problem 9 involves a photograph.)

9. DOOR FRAME—The diagram shows the top of a door frame.  $\angle HGD$  and  $\angle HGF$  are right angles,  $m \angle DGB = 21^\circ$ ,  $m \angle HBG = 55^\circ$ ,  $\angle DGB = \angle CGF$ , and  $\angle HBG \cong \angle HCG$ . What is  $m \angle HGC$ ?
- $21^\circ$
  - $111^\circ$
  - $69^\circ$
  - $159^\circ$



10. GARDEN—Jim wants to lay bricks end-to-end around the border of the garden as shown below. Each brick is 10 inches long. Which expression can be used to find the number of bricks Jim needs?

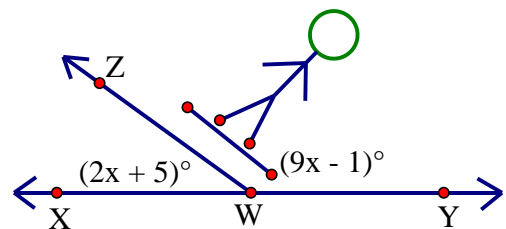
- $26\pi \div \frac{10}{12}$
- $52\pi \div \frac{12}{10}$
- $13^2\pi \bullet \frac{12}{10}$
- $13\pi \bullet \frac{10}{12}$



11. AREA—The points A (-4, 0), B (0, 2), C (4, 0) and D (0, -2) are plotted on a coordinate grid to form the vertices of quadrilateral. What is the area of quadrilateral ABCD?
- 8 square units
  - 16 square units
  - 20 square units
  - 32 square units

12. SKATEBOARDING—As shown in the diagram, a skateboarder tilts one end of a skateboard. What is  $m \angle ZWX$ ?

- $11^\circ$
- $16^\circ$
- $21^\circ$
- $37^\circ$



## Constructions

Construction tools = technical drawings that require the use of a compass and a straight edge.

1. Compass—the tool used to \_\_\_\_\_ and \_\_\_\_\_

- **No** numbers are used for measurements in a construction.
- **No** curves are ever drawn ‘freehand’ in a construction.

2. Straight edge—the tool used for all \_\_\_\_\_

- **No** segments are drawn ‘freehand’ in a construction.
- **No** guessing as to where something should be in a construction.

**Practice** each of these constructions until you are comfortable with them.

**Include a copy of each construction with this packet**—on this paper or separate paper.

- Section a circle into 6 congruent pieces.
  1. Draw a circle using the compass.
  2. Without changing the setting of your compass, align the compass with the edge of the circle.
  3. Make a mark on the circle with the other end of the compass.
  4. Move the compass to the mark you just made.
  5. Make another mark using the compass.
  6. Continue until you have gone all the way around the circle—you should end exactly where you started.



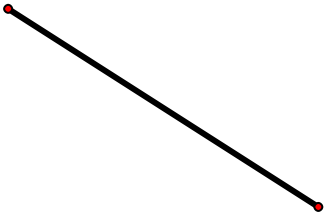
- Construct a  $60^\circ$  angle
  1. Construct a circle and section it into 6 congruent pieces (see above)
  2. Connect every other point.
  3. Measure the angles—they should be exactly  $60^\circ$



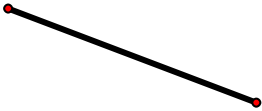
- Construct a  $120^\circ$  angle
  1. Construct a circle and section it into 6 congruent pieces (see above)
  2. Connect 3 consecutive points.
  3. Measure the angles—they should be exactly  $120^\circ$



- Copy a segment
  1. Draw a segment that is longer than the one you are copying.
  2. Use the compass to measure the segment (set the compass to the length of the segment)
  3. Align the compass with the endpoint of the copy you are making.
  4. Make a mark with the compass on the segment at the other endpoint.



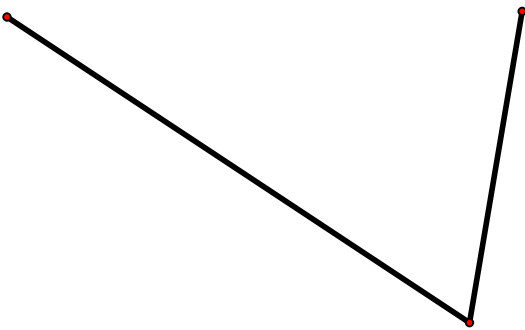
- Double the length of a segment
  1. Draw a segment that is longer than the one you are copying.
  2. Use the compass to measure the segment (set the compass to the length of the segment)
  3. Align the compass with the endpoint of the copy you are making.
  4. Make a mark with the compass on the segment at the other endpoint.
  5. Make a second mark with the compass from the previous compass marking.



- Bisect a segment
  1. Set your compass at 'a little more than half the segment'.
  2. Make 2 arcs (one above the segment, one below) from the first endpoint.
  3. Make 2 arcs from the other endpoint—these need to be long enough to intersect the first pair of arcs.
  4. Connect the 2 intersections—this segment will intersect the first one at its midpoint.



- Copy an angle
  1. Draw a ray to work from.
  2. Make 2 arcs on the original angle at a set distance from the vertex.
  3. Make 2 arcs on the copy at this same distance from the vertex—make the second one a bit longer than you think is necessary.
  4. **CHANGE THE COMPASS SETTING** so that it shows the distance between the compass marks on the original angle.
  5. Transfer this measurement to the copy—this mark must intersect the second arc from earlier.
  6. Connect the intersection to the endpoint of the original ray.



- Bisect an angle
  1. Make 2 arcs on the angle at a set distance from the vertex.
  2. Make one arc from each of these intersections—make sure that these arc intersect.
  3. Connect the intersection to the vertex of the angle.

